1

Dr. Josef Stefan I.

Mechanik

I. S. 90/91 M Smoluchowski

9'4
1'4

8

9
18'4 } 9'4
10'4 } 8

$\frac{9'4}{8} = 1'2$

10
24 } 14
18'8 } 5'2

14 : 5'2 = 2'8
46

53'902 : 50'75 = 1'0621
3152
107
6

4'30
4'59

4'59 : 4'30 = 1'067
29
3

53'92 : 50'75 = 1'062
317
13
3

2

5 : x = 9 : 1 BJ

x = 5 : 9 = 0·5555

16/10

[illegible] 8 20 [illegible]

[illegible] 20

[illegible]

[illegible]

[illegible] = calorie = [illegible]

[illegible]

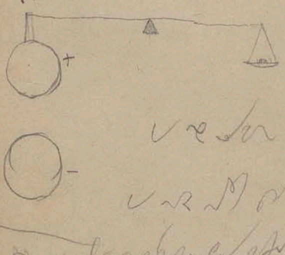
[illegible]

[illegible] m m² m³ [illegible] 43 [illegible]

[illegible]

[illegible] 1 = [illegible] [[illegible] 10]

[illegible]



[illegible]

[illegible] = 20 [illegible]

[illegible]

[illegible]

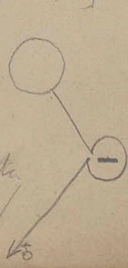
[illegible] [[illegible] 10] [illegible]

[illegible]

[illegible]

[illegible] = [illegible] 120 [illegible] etc

[illegible] 120 < [illegible]



[illegible]

Übersicht über Stefans Vorlesungen.

[illegible] Statik [illegible] Dynamik

[illegible]

[illegible]

Differential-R. von Newton [illegible] Leibnitz

[illegible]

Discussion [illegible]:

$s = at^2 + bt + c$ [illegible]

$s = a \sin \alpha t$ [illegible]

[illegible] $f = \frac{d\left(\frac{mv^2}{2}\right)}{ds}$

[illegible] $s = \int \varphi(t)\, dt$

[illegible]

[illegible] $v = \int \varphi(t)\, dt$

[illegible]

[illegible]

$\int dt = \int \chi(s)\, ds$

[illegible] $v = a\sqrt{s}$

7/11

[illegible]

[illegible] Galilei [illegible]:

1.) [illegible] —

[illegible]

2). [illegible]

[illegible] Galilei

[illegible]

[illegible] abs. [illegible] sp. [illegible] etc. m = [illegible]

[illegible]

11/11 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

12/11 [illegible]

[illegible]

[illegible]

Kinematik [Geometrie [illegible]]

[illegible]

1). [illegible]

2). [illegible]

3). [illegible]

[illegible] centrip. [illegible] 13/11

[illegible] Project.

14/11

[illegible] 3 Keppler'schen [illegible]

[illegible] Centrale [illegible]

[illegible] Gravitations [illegible] Keppler'schen [illegible] 18/11

[illegible]

[illegible] Ell. [illegible] RV [[illegible]] [illegible] 20/11

[illegible]

[illegible] Gravit. 21/11

[illegible] RV ; [illegible] RV.

[illegible] Polar-Coordinaten.

[illegible] RV. 25/11

[illegible] Gravitations [illegible] Keppler'schen [illegible] Polarcoord.

26/11 [illegible] etc. [illegible]

[illegible]

[illegible]

[illegible] [[illegible]]

1). [illegible] 2). [illegible]

[illegible]

I. [illegible]

II [illegible]

27/11 a). Hypoth. $f(u) = u$
b). – $f(u) = u^2$ } [illegible]

28/11 [illegible] etc. [illegible]

" " " [illegible] a). Hyp. $f(u) = u$; [illegible] etc.

2/12 b). Hyp. $f(u) = u^2$; maxim. [illegible]

3/12 } [illegible] R. S. [illegible] Newton'schen Gravit. / [[illegible] Polarcoord]

4/12 } Discussion [illegible] – Ell – Hyp – Par [illegible]

Constr. [illegible]

9/12 Kraftfunction

[illegible]

potentielle u. kinetische Energie ; [illegible] Energie

10/12 [illegible]

[illegible]

[illegible]

12/12 [illegible]

[illegible] Centralk. [illegible]

16/12 [illegible]

[illegible]

[illegible]

17/12 [illegible]

[illegible]

[illegible]

7/1 [illegible]

[illegible]

8/1 [illegible]

9/1 [illegible]

13/1 [illegible]

14/1 [illegible]

Ellipsen [illegible]

1). [illegible] Prinz. [illegible]

15/1 2). [illegible] = Ell.

16/1 Pr. [illegible] virt. [illegible]

[illegible] ; [illegible] (2√) ; [illegible] ‖ 2B. [illegible]

20/1 d'Alembert Pr. 2B. [illegible]

Pr. d. v. V. [illegible]

21/1 Lagrange' [illegible] 2B. [illegible] Polar. ; [illegible]

22/1 [illegible]

Σ [illegible]

23/1 [illegible]

([illegible])

[illegible], Res.

27/1 [illegible]

28/1

29/1 [illegible] Th.

30/1 [illegible] Δ, ⏢, emp. [illegible]

3/2 [illegible]

4/2 [illegible]

6/2 [illegible]

12/2 [illegible]

[illegible] red. [illegible]

13/2 [illegible] [[illegible], [illegible], Prisma, Ellipsoid,] [illegible]

17/2 Revers. [illegible], LK [illegible]

18/2 [illegible]

19/2 [illegible]

20/2 [illegible]

24/2 [illegible]

25/2 [illegible]

[illegible] Aroma

26/2 [illegible]

27/2 [illegible]

[illegible]

3/3 [illegible]

LK — — —

[illegible]

4/3 [illegible]

[illegible] = 1 m

[illegible]

[illegible]

$\frac{[illegible]}{10,000000}$ = m [illegible]

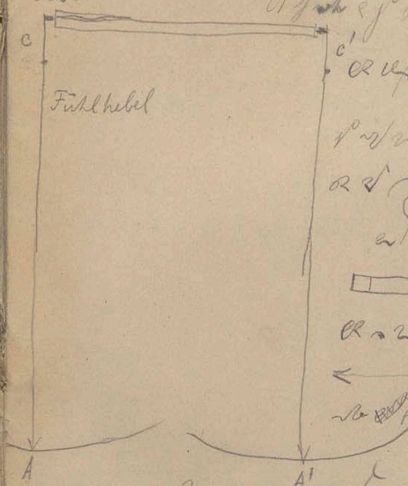
[illegible]

443‴296 Pariser Linien = 1 m

[illegible]

[illegible] Pt [illegible]

[illegible]

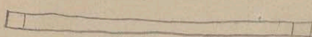


[illegible] Prototype

[illegible] Pt [illegible] Pt Ir [illegible]

[illegible]

[illegible]



[illegible]

[illegible] Comparatore

2 mikrons [illegible] [[illegible]]

[illegible]

[illegible]

[illegible] [[illegible]]

[illegible shorthand]

[illegible shorthand]

17/10

[illegible shorthand heading]

[illegible shorthand]

[illegible shorthand]

[illegible shorthand] Nonius



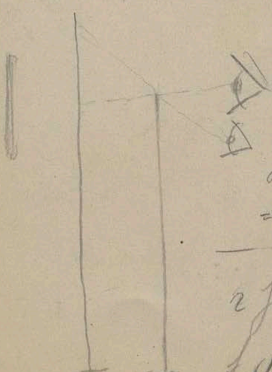
[illegible shorthand]

[illegible shorthand] decimale

[illegible shorthand] Nonius [illegible shorthand]

[illegible shorthand]

[illegible shorthand]



[illegible shorthand]

[illegible shorthand]

[illegible shorthand]

[illegible shorthand]

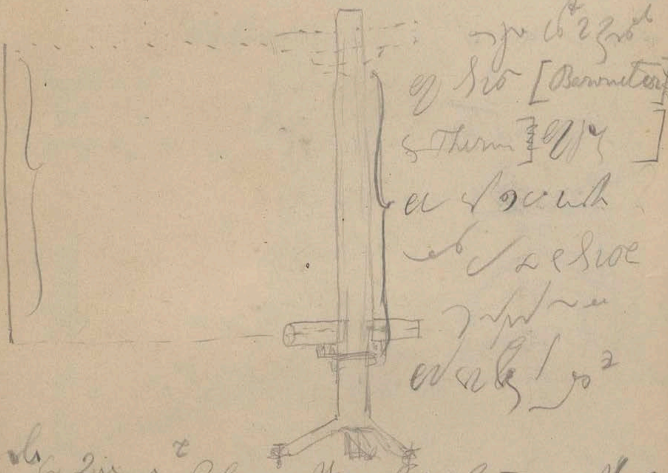
[illegible shorthand]

[illegible shorthand]

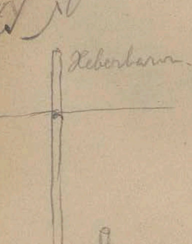
Kathetometer [illegible]

[illegible]

[illegible] = 120 [illegible]



[illegible] 2 [illegible]



[illegible] 2 [illegible]

[illegible]

Mikrometer[illegible]

[illegible]

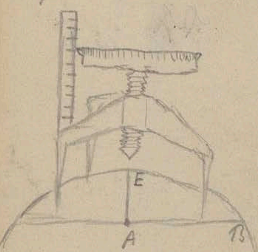
[illegible] [illegible]meter [illegible]

[illegible]

[illegible]

[illegible] comparator [illegible]

Sphärometer [illegible]



[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] Sphärometers [z Wildt]

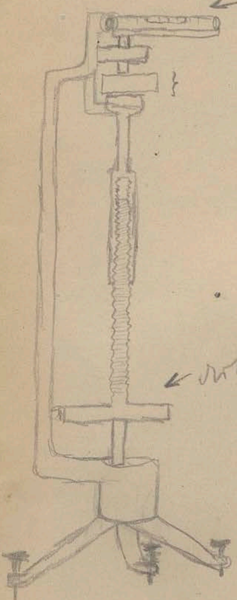
[illegible]

[illegible]

[illegible] Libelle [illegible]

[illegible]

[illegible]



21/10

[illegible] 1m [illegible] 0°

[[illegible] 12 R [illegible]]

[illegible]

$$l = l_0(1+\alpha t)$$

$$\frac{L}{\lambda} = n$$

$$\lambda = \lambda_0[1+\alpha t]$$

$$\frac{L}{\lambda_0[1+\alpha t]} = n$$

$$\frac{L}{\lambda_0} = n[1+\alpha t]$$

$$L_t = L_0[1+\beta t]$$

$$L_0 = \frac{L_t}{1+\beta t} = n\lambda_0\frac{[1+\alpha t]}{[1+\beta t]}$$

$\lambda =$ [illegible]

$L =$ [illegible]

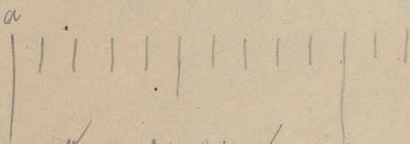
$\alpha =$ [illegible] coeff. [illegible]

$\beta =$ " " [illegible]

[illegible]

[illegible]

[illegible] – [illegible] Rattekometer



[illegible]

[illegible]

[illegible]

[illegible] z.B. 5 [illegible]

[illegible]

[illegible] [– [illegible] 3 [illegible]]

[illegible]

[illegible]

[illegible]

[illegible] Thermom. scalen [illegible] z.B. mm [illegible]

[illegible]



[illegible] [z.B. [illegible] Remontoir Uhren].

[illegible] [[illegible]]

[illegible]

[illegible]

[illegible]

Massen[illegible] [illegible] Gewichts[illegible]

1 [illegible] $= 1$ [illegible] cm^3 [illegible] $4°C = 1g$

[illegible] cm^3 [illegible]

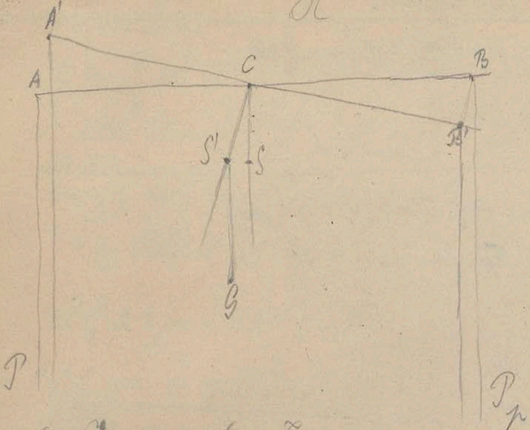
[illegible]

kg [illegible]

[illegible]

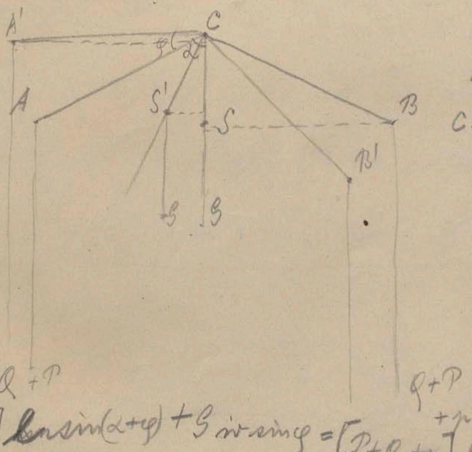
[illegible] [[illegible]]

S = [illegible]



CS [illegible]

[illegible] etc.



$CAS = \alpha$

$S'CS = \varphi = A'CA$ etc.

$AC = BC = A'C = B'C = l$

$CS' = CS = w$

$\varphi = ?$

$$[P+Q]\, l \sin(\alpha+\varphi) + G w \sin\varphi = [P+Q+p]\, l \sin(\alpha-\varphi)$$

$$~~[P+Q]\sin\alpha\cos\varphi\, l~~ + [P+Q]\cos\alpha\sin\varphi\, l + G w \sin\varphi = ~~[P+Q+p]~~\, l \sin\alpha\cos\varphi - [P+Q+p]\, l \cos\alpha\sin\varphi$$

$$2[P+Q]\, l \cos\alpha\sin\varphi + G w \sin\varphi = p\, l \sin\alpha\cos\varphi - p\, l \cos\alpha\sin\varphi$$

$$\operatorname{tg}\varphi = \frac{p l \sin\alpha}{g w + \{2[\sigma + \varphi] - p l\}\cos\alpha}$$

$$= \frac{p l \sin\alpha}{g w + [2\sigma + 2\varphi - p l]\cos\alpha}$$

[illegible] tg proport [illegible]

[illegible]

$$\frac{\operatorname{tg}\varphi}{p} = [illegible] = \frac{l \sin\alpha}{g w + [2\sigma + 2\varphi - p l]\cos\alpha}$$

[illegible], $\alpha = 90^\circ$ [illegible]

[illegible]

[illegible]:

$$\frac{\operatorname{tg}\varphi}{p} = \frac{l}{g w}$$

[illegible]

[illegible]

[illegible]

[illegible]

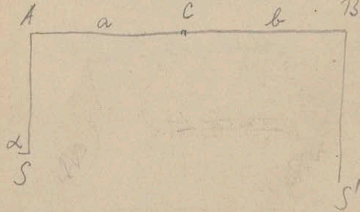


23/20

[illegible]

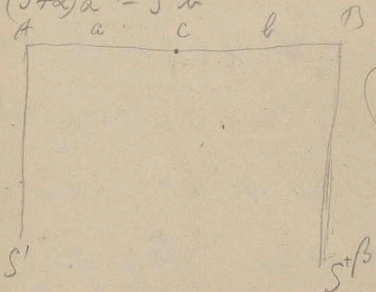
[illegible]

[illegible]



[illegible]

$(S+\alpha)a = S'b$



$S'a = (S+\beta)b$

[illegible]

$$\left.\begin{aligned}(S+\alpha)a &= S'b\\ (S+\beta)b &= S'a\end{aligned}\right\} \quad (S+\alpha)(S+\beta) = S'^2$$

$$S^2 + S(\alpha+\beta) + \alpha\beta = S'^2$$

$$\alpha S + \beta S + \alpha\beta = S'^2 - S^2$$

$$= (S'-S)(S'+S)$$

$$S'-S = \frac{S}{S+S'}(\alpha+\beta) + \frac{\alpha\beta}{S+S'}$$

$$\frac{S}{S+S'} \neq \frac{1}{2}$$

[illegible]

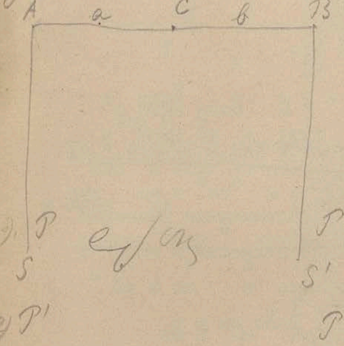
$$S'-S = \frac{\alpha+\beta}{2}$$ [=?]

[illegible]

[illegible]

$(S+p)a = Sb$

[illegible]



$$Pa = P'b$$

~~$P''a = P'b$~~

$$Pb = P''a$$

$$P^2ab = P'P''ab$$

$$P = \sqrt{P'P''}$$

[illegible]

[illegible] Chemie [illegible]

[illegible] s.B. sp. S. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$200\,g = \Sigma$

$\Sigma = 200$

100	50	20	10	10	5	2	1	1	1
	a	b	c_1	c_2	d	e	f_1	f_2	f_3
						2	2		1

[illegible]

[illegible]

$$100 = a + b + c_1 + c_2 + d + e + f_1 + f_2 + f_3 + \alpha$$

$$a = b + c_1 \; \sim\sim\sim \; + f_3 + \beta$$

$$100 - a = a + \alpha - \beta$$

$$100 - \alpha + \beta = 2a$$

$$a = 50 - \frac{\alpha - \beta}{2}$$

$$50 - \frac{\alpha - \beta}{2}$$

$$b + c_1 + c_2 + d + e + f_1 + f_2 + f_3 = A$$

$$b = c_1 + c_2 + \gamma$$

$$b = c_1 + d + e + f_1 + f_2 + f_3 + \delta$$

$$b = c_2 + \; \sim\sim\sim \; + f_3 + \varepsilon$$

8 b c_1, c_2, $[e + f_1 + f_2 + f_3]$, [illegible]

$$d + e + f_1 + f_2 + f_3 = B$$
$$d = e + f_1 + f_2 + f_3 + \eta$$

$$2d - \eta = B$$
$$d = \frac{B + \eta}{2} \quad \text{etc.}$$

1. Fig.:

5 2 2 1

$d \quad e_1 \quad e_2 \quad f$

$$e_1 + e_2 + f = C$$
$$e_1 = e_2 + \lambda$$

[illegible]

e:

$$e_1 + e_2 + f = C$$
$$e_1 = e_2 + \lambda$$
$$e_1 = 2f + \mu$$

---

etc.

[illegible] e f [illegible] [zur Kopie machen]

zB. ~ e

24/10

[illegible]

1. [illegible] Praecessions[illegible]

2. [illegible]

[illegible]



z.B. [illegible] $\frac{4}{10}$ [illegible]

[illegible] $\frac{4}{10}$ [illegible]



[illegible] cgm

[illegible]

[illegible] [[illegible]]

[illegible]

[illegible]

z.B.

1l [illegible] 1kg [illegible]

1l [illegible] 1·293 g | [illegible] $\frac{1·293}{8·2}$ g

[illegible]
[illegible]
P A [illegible]

$$P - V\lambda = A - v\lambda$$

$$V = \frac{P}{S} \qquad v = \frac{A}{s}$$

$$P - \frac{P\lambda}{S} = A - \frac{A\lambda}{s}$$

$$P\left[1 - \frac{\lambda}{S}\right] = A\left[1 - \frac{\lambda}{s}\right]$$

$$P = A \cdot \frac{1 - \frac{\lambda}{s}}{1 - \frac{\lambda}{S}}$$

$\lambda = 0{\cdot}001293$ [illegible] $\frac{\text{Luft}}{\text{Wasser}} = \frac{1}{770}$

[illegible] temp. [illegible] Barom[illegible]

λ [illegible] $0°$ [illegible] 760 mm

λ' bei b' $\quad \lambda' = \frac{b'}{760}\lambda \quad [$ [illegible] $0°]$

$$\lambda' = \frac{b'}{760}\lambda \frac{1}{[1 + \alpha t]}$$

t [illegible] $\alpha = \frac{1}{273} = 0{\cdot}000366$

[illegible]

[illegible]

b' { [illegible] b_1
[illegible] b_2

$$\lambda' = \frac{b_1}{760}\lambda\frac{1}{1+\alpha t} + \frac{b_2}{760}\lambda\frac{1}{1+\alpha t}\cdot 0{\cdot}62$$

[illegible]

$$= \frac{b_1 + b_2\, 0{\cdot}62}{760}\,\frac{\lambda}{1+\alpha t}$$

[illegible] Quarz [illegible]

[illegible]

[illegible]: e sp. 3. [illegible]

[illegible] Pyknometer



S [illegible] ↑ ↗

$$S\cdot\left(1 - \frac{\lambda}{\rho}\right) = [illegible]$$ [illegible]

1 [illegible]

$$S\left(1-\frac{\lambda}{\rho}\right) = A\left(1-\frac{\lambda}{a}\right)$$

$a =$ sp. 3. [illegible]

$A =$ [illegible]

2 [illegible]

$$S\left(1-\frac{\lambda}{\rho}\right) + V[\sigma-\lambda] = A'\left(1-\frac{\lambda}{a}\right)$$

[illegible]

$$S\left(1-\frac{\lambda}{\rho}\right)+v[s-\lambda]=A''\left[1-\frac{\lambda}{a}\right]$$

$$v[\sigma-\lambda]=[A'-A]\left[1-\frac{\lambda}{a}\right]$$

$$v[s-\lambda]=[A''-A]\left[1-\frac{\lambda}{a}\right]$$

$$\underbrace{\frac{s-\lambda}{\sigma-\lambda}}_{\text{[illegible]}}=\frac{A''-A}{A'-A}$$

[illegible]:

$$s=\lambda+[1-\lambda]\,13{\cdot}6$$

$$\lambda=0{\cdot}00129 \,.\, 126$$

```
    258
    774
---------
0·016254
```

$$s=13{\cdot}6-\underbrace{0{\cdot}0163}_{\text{[illegible]}}$$

[illegible] 0·001 ....

28/10

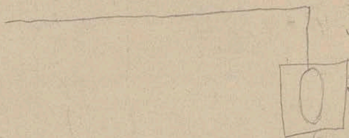
$S \quad S'$ $\qquad \dfrac{S-\lambda}{S'-\lambda}$ = [illegible] [[illegible]]

[illegible]

$a =$ [illegible]

V. hydrost. Wage

$P =$ [illegible]



$P - v\lambda = A\left(1-\frac{\lambda}{a}\right)$

$P - vS = B\left(1-\frac{\lambda}{a}\right)$

$P - vS' = C\left(1-\frac{\lambda}{a}\right)$

[illegible]

$v(S-\lambda) = (A-B)\left(1-\frac{\lambda}{a}\right)$

$v(S'-\lambda) = (A-C)\left(1-\frac{\lambda}{a}\right)$

$\dfrac{S-\lambda}{S'-\lambda} = \dfrac{A-B}{A-C}$ [illegible]

[illegible]

$P - v\lambda = (1-a)A$

$P - vS' = C(1-a)$

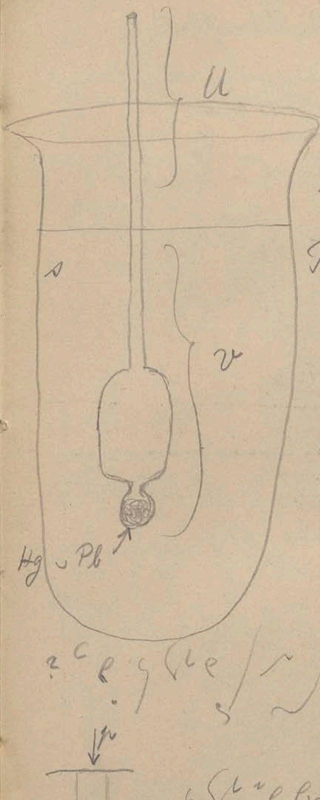
$v(S'-\lambda) = (A-C)(1-a)$

$P = v\Sigma$?

$v(\Sigma-\lambda) = A(1-a)$

$\dfrac{\Sigma-\lambda}{S'-\lambda} = \dfrac{A}{A-C}$

[illegible] : Die Scalenaräometer



[illegible]

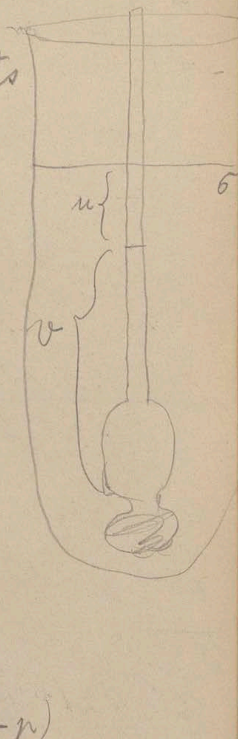
$$P = v s$$

[illegible]

$$P=(v+u)\sigma$$

$$\frac{\sigma}{s} = \frac{v}{v+u}$$

$$\frac{s}{\sigma} = \frac{v+u}{v}$$



[illegible]



[illegible] $(p'-p)$

[illegible] Aräom. [illegible]

[illegible]

[illegible]:

$$P - U\lambda = V s \qquad P - (U-u)\lambda = (V+u)\sigma$$

$$u\lambda = (V+u)\sigma - Vs$$

$$Vs = (V+u)\sigma - u\lambda - V\lambda + V\lambda$$

$$= (V+u)(\sigma-\lambda) + V\lambda$$

$$V(s-\lambda) = (V+u)(\sigma-\lambda)$$

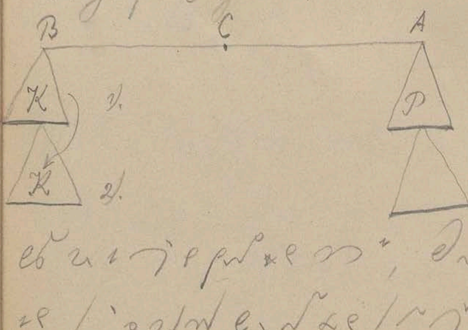
$$\frac{s-\lambda}{\sigma-\lambda} = \frac{V+u}{V}$$

[illegible] [[illegible] centrif. [illegible] Abplatt.]

[illegible]

[illegible] [[illegible]]

[illegible].



z.B. 2 [illegible] kg.

[illegible] K [illegible]

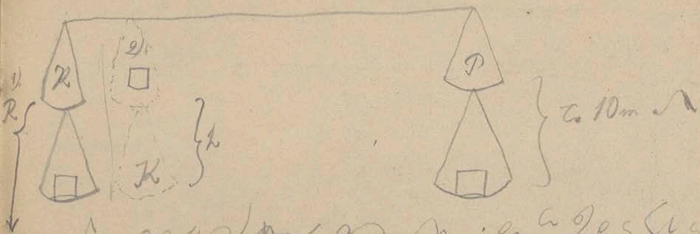
[illegible]

[illegible].

[illegible]

[illegible].

Jolly [illegible]



[illegible]

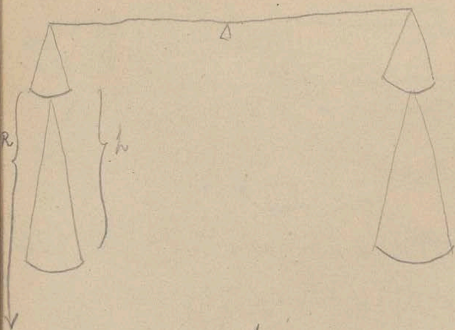
[illegible]

$$A' - A = p$$

$$A' : A = \frac{1}{R^2} : \frac{1}{(R+h)^2}$$ ~~[illegible]~~

[illegible] Jolly [illegible]

29/10



$A : A' = \frac{1}{R^2} : \frac{1}{(R-h)^2}$

$A' = \frac{AR^2}{(R-h)^2} = A\frac{1}{\left(1-\frac{h}{R}\right)^2} = A\left(1-\frac{h}{R}\right)^{-2}$

$\doteq A\left(1+\frac{2h}{R}\right)$

$A'-A = A\frac{2h}{R}$ = [illegible]

$A'-A = A\frac{2h\frac{\pi}{2}}{\pi\frac{R}{2}} = A\frac{\pi h}{10^7}$

für $h = 1$ m, $A = 1$ kg: $A'-A = \frac{\pi}{10^7} = \frac{0.314}{10^6}$

$= 0.314$ mg

[illegible]

[illegible] Jolly: $h = 20$ m, $A = 5$ kg

$A'-A = \frac{5 \cdot 20 \cdot 0.314}{10^6} = \frac{314}{10^6} = 31.4$ mg

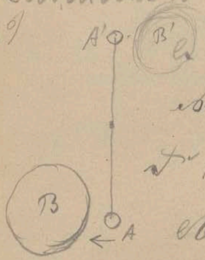
[illegible]

2. [illegible]

[illegible]

[illegible]

Cavendish



[illegible] [Torsions[illegible]] [illegible]

[illegible] etc.

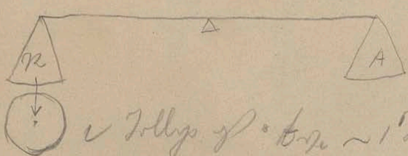
[illegible] 5.6 [illegible]

[illegible] Paris [illegible] Cornu

[illegible] z.B. Pendelschwingungen [illegible]

[illegible]

[illegible] etc.



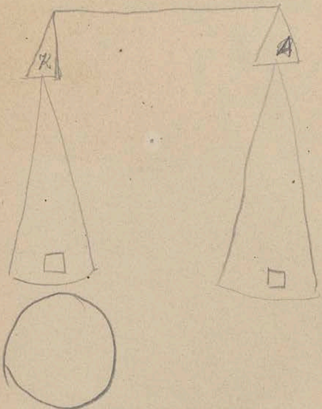
$\frac{M[illegible]}{R^2} : \frac{m[illegible]}{[illegible]} : K : p$

[illegible] Jollys [illegible] 6000 kg [illegible] $\frac{1}{2}$ m

[illegible] 5 kg Hg

[illegible] $\frac{1}{2}$ mg

[illegible]

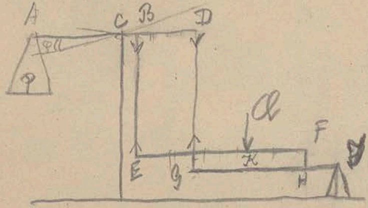


[illegible] = 31 mg

" " [illegible] = 0·5 mg

3 0/00

[illegible]



[illegible]

B [illegible]

E [illegible]

[illegible]

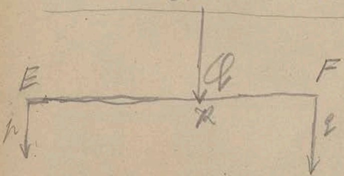
$$P\,\overline{AC} = p\,CB + p'\,CD$$

$$P\,AC = Q\frac{KF}{EF}\,CB + q\,\frac{HJ}{GJ}\,CD$$

$$= Q\frac{KF}{EF}\,CB + Q\,\frac{HJ}{GJ}\,CD - Q\frac{KF}{EF}\,\frac{HJ}{GJ}\,CD$$

[illegible]

[illegible]



$Q = p + q$

$p\,KE = q\,KE$

$p\,KE = (Q - p)\,KF$

$p\,EF = Q\,KF$

[illegible]

$p'\,GJ = q\,HJ$

[illegible]

[illegible]

~~[illegible]~~ $Q \frac{KF}{EF}\left[CB - \frac{HJ}{GJ} CD\right] = 0$ [illegible]

$$CB = \frac{HJ}{GJ} CD$$

$$\frac{CB}{CD} = \frac{HJ}{GJ}$$

$$CB : CD = HJ : GJ$$

[illegible]

$$P \overline{AC} = Q \frac{HJ}{GJ} CD = Q\, CB$$

[illegible]

[illegible] $CB : CD = 1 : 10$ [illegible]

[illegible]:

A [illegible]

$AA' = AC\,\varphi$

$BB' = CB\,\varphi$

$DD' = CD\,\varphi$

$EE' = BB'\varphi = CB\,\varphi$

$GG' = DD' = CD\,\varphi$

$HH' = \frac{HJ}{GJ} GG'$

$FF' = HH' = \frac{HJ}{GJ} GG'$

$FF' = \frac{HJ}{GJ} CD\,\varphi$

$FF' = CB\,\varphi = EE'$

[illegible]

[illegible] vorstellen [illegible]:

$PAA' - Q\,KK' = 0$ [illegible]

$PAA' = Q\,KK'$

[illegible]

[illegible] Producte [illegible]

[illegible]

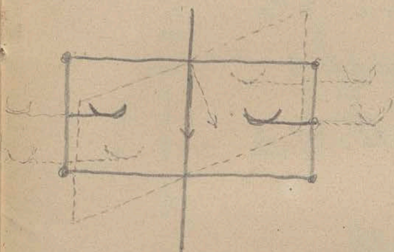
[illegible] Prod. [illegible]

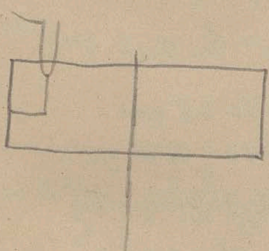
[illegible]

[illegible] Quintenz in Strassburg constr. 1825

– [illegible] Schönemann

– [illegible] Roberval'sche Parallelogramm





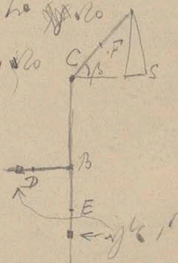
f101 :

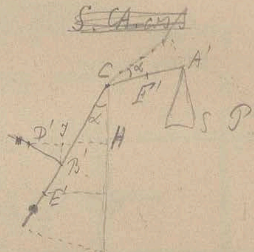


D = Schwer[illegible]

E = [illegible]

F = [illegible]





$$S.CA.\cos\beta + g.CF.\cos\beta = g'\,BD$$

$$(S+P)\,CA'\cos(\beta-\alpha) + g\,CF\cos(\beta-\alpha) = g'(D'J + JH) + G.CE'\sin\alpha$$

$$D'J = D'B'\cos\alpha$$

$$JH = CB'\sin\alpha$$

$$(S+P)CA\cos(\beta-\alpha) + g\,CF\cos(\beta-\alpha) = g'(DB\cos\alpha + CB\sin\alpha) + G.CE\sin\alpha$$

$$(S+P)CA\cos\beta\cos\alpha + (S+P)CA\sin\beta\sin\alpha + g\,CF\cos\beta\cos\alpha + g\,CF\sin\beta\sin\alpha =$$

$$P\,CA\cos(\beta-\alpha) + S.CA\sin\beta\sin\alpha + g\,CF\sin\beta\sin\alpha = g'\,CB\sin\alpha + G.CE\sin\alpha$$

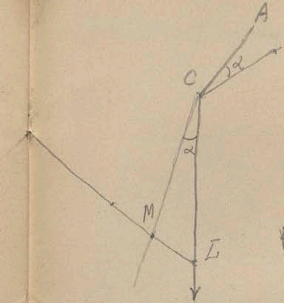
$$P \cdot CA \cos(\beta-\alpha) = (g'CB + G\,CE - g\,CF \sin\beta - S \cdot CA \sin\beta)\sin\alpha$$

$$P = K \frac{\sin\alpha}{\cos(\beta-\alpha)}$$

[illegible]

[illegible]

[illegible] Scale [illegible]



[illegible] LM [illegible]

[illegible]

$$\measuredangle LM : CL = \sin\alpha : \cos(\beta-\alpha)$$

$$\cos(\beta-\alpha) = \sin(90+\beta-\alpha)$$

$$90+\beta-\alpha = \measuredangle CML$$

$$\measuredangle CLM = 180 - CML - \alpha$$

$$= 90 - \beta$$

[illegible] L [illegible] $(90-\beta)$ [illegible]

[illegible]



[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

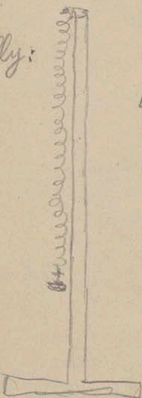
[illegible]

[illegible]

[illegible]

[illegible]:

Jolly:



[illegible] scala [illegible]

[illegible]

[illegible]

[illegible] Statik [illegible] Dynamik –

[illegible]

[illegible]

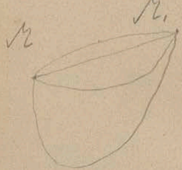
[illegible]

[illegible] Lagrange

[illegible]

[illegible]

[illegible]



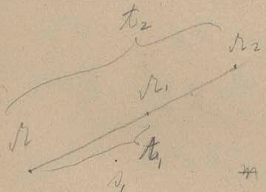
[illegible]

[illegible]

[illegible]

[illegible]

[illegible]



[illegible]

[illegible]

[illegible]

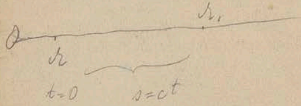
$s_1 \quad t_1$

$s_2 \quad t_2$

$$\frac{s_2 - s_1}{t_2 - t_1} = \frac{s_1}{t_1} = \frac{s_2}{t_2} = \text{[illegible]}$$

$\frac{s}{t} = c \qquad s = ct \qquad Or_1 = OM + MM_1$

$s = a + ct$ [illegible]



[illegible]:

[illegible]

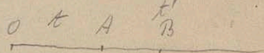
[illegible]



[illegible]

[illegible]

[illegible]



$\frac{OB-OA}{t'-t}$ [illegible]

continuirlich [illegible]

$s = f(t)$

$s' = f(t')$

$\underbrace{\frac{s'-s}{t'-t}}_{\tau} = \frac{f(t')-f(t)}{t'-t} \qquad \Big| \quad t'-t=\tau$

$$\frac{s'-s}{\tau} = \frac{f(t+\tau)-f(t)}{\tau}$$

[illegible] $= \lim \frac{f(t+\tau)-f(t)}{\tau}$

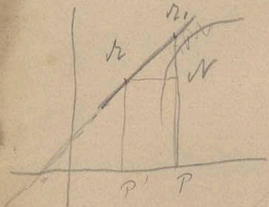
$s = at^2$

$tg\,\alpha = \lim \frac{a(t+\tau)^2 - at^2}{\tau}$

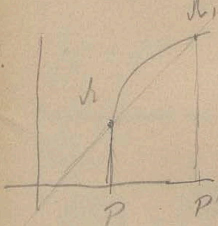
$= \lim \frac{at^2 + 2at\tau + a\tau^2 - at^2}{\tau} = \lim (2at + a\tau)$

$= 2at$

[illegible] Fluxions [illegible] Differential R. [illegible] lim [illegible]



$\frac{M'P - MP'}{PP'} = tg\,\alpha$



$\frac{M'P' - MP}{PP'}$ | M' [illegible] M

$y = f(x)$

$y' = f(x + \xi)$

$tg\,\alpha = \lim \frac{f(x+\xi) - f(x)}{\xi}$ } [illegible] Leibnitz [illegible] Diff. R. [illegible]

$$\operatorname{tg}\alpha = \lim \frac{f(t+\tau) - f t}{\tau}$$

$$= \frac{d f(t)}{d t} = \frac{d s}{d t}$$

$$\operatorname{tg}\alpha = \frac{d f(x)}{d x} = \frac{d y}{d x}$$

5/11

$s = c\,t$

$s = a\,t^2$

$\frac{ds}{dt} = 2at = u$

$u_1 = 2at_1$

$u_2 = 2at_2$

$u_2 - u_1 = 2a(t_2 - t_1)$

$\frac{u_2 - u_1}{t_2 - t_1} = 2a$ = [illegible] [$a\ t_2 - t_1 = 1\,s$]

= [illegible]

[illegible]

[illegible]

$f = \frac{du}{dt} = 2a$

$s = f(t)$

$u = \frac{ds}{dt} = \frac{df(t)}{dt} = f'(t)$

$u_1 = f'(t_1)$

$u_2 = f'(t_2)$

$\frac{u_2 - u_1}{t_2 - t_1} = \frac{f'(t_2) - f'(t_1)}{t_2 - t_1}$

[illegible]

[illegible]

[illegible]

~~$\frac{f'(t_1+\tau)-f't_1}{t_2-t_1} = \tau$~~ $= \frac{f'(t_1+\tau)-f'(t_1)}{\tau}$

$$= \lim \frac{f'(t_1+\tau)-f'(t_1)}{\tau}$$

[illegible] t_1 [illegible]

conventionelles Zeichen

$$= \frac{d\,f'(t)}{dt} = \frac{du}{dt} = \frac{d^2 s}{dt^2}$$

$$= \frac{d}{dt}\cdot\frac{ds}{dt} =$$

[illegible]

$s = at^2 + bt + c$ [illegible]

$t = 0$

$s_0 = c$ [illegible]

$u = \frac{ds}{dt} = 2at + b$

$t = 0$

$u_0 = b$

$f = \frac{du}{dt} = 2a$ [illegible]

[illegible] $t=0$ [illegible]

[illegible] $t=0$

[illegible]

$s = \alpha t^2 - \beta t$

$u = \frac{ds}{dt} = 2\alpha t - \beta$ [illegible] $\beta = 2\alpha t$ [illegible] $= 0$ [illegible]

$t_1 =$ [illegible]

$0 = 2\alpha t_1 - \beta$

$t_1 = \frac{\beta}{2\alpha}$

$s_1 = \alpha \frac{\beta^2}{4\alpha^2} - \beta \frac{\beta}{2\alpha} = -\frac{\beta^2}{4\alpha}$

[illegible]

$s = -\alpha t^2 + bt$

$\frac{ds}{dt} = u = -2\alpha t + b$

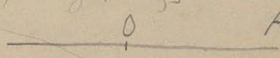
$t = 0$

$s = 0$

$u = b$

[illegible]

$t_1 = \frac{b}{2\alpha}$ [illegible] $= 0$ [illegible]



[illegible] 0

$s = 0$

$-\alpha t_2^2 + b t_2 = 0$ I $t_2 = \frac{b}{\alpha}$ II $t_2 = 0$

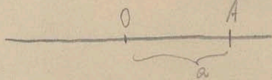
$u_2 = -2\alpha \frac{b}{\alpha} + b$

$= -b$

$s = a \sin \alpha t$

$t = 0$
$s = 0$

$\alpha t_1 = \frac{\pi}{2}$
$s = a$



$\alpha t_2 = \pi$
$s = 0$

$\alpha t_3 = \frac{3\pi}{2}$
$s = -a$

$\alpha t_4 = 2\pi$
$s = 0$

[illegible]

$t_4 = \frac{2\pi}{\alpha} =$ [illegible]

$\frac{ds}{dt} = \alpha a \cos \alpha t = u$

t_1	$u = 0$
t_2	$u = -\alpha a$
t_3	$u = 0$
t_4	$u = \alpha a$

$f = \frac{du}{dt} = -\alpha^2 a \sin \alpha t = -\alpha^2 s$

[illegible]

[illegible]
[illegible]

$u = \alpha \cdot \sqrt{a - s^2}$

6/11

$s = a \sin \alpha t$

$u = \frac{ds}{dt} = \alpha a \cos \alpha t = \alpha \sqrt{a^2 - s^2}$

$u = \varphi(s)$

$t \quad u$

$t' \quad u'$ | $f = \lim \frac{u' - u}{t' - t}$

$t \quad u - - s$

$t' \quad u' - - s'$

$u' - u = \varphi(s') - \varphi(s)$

$\frac{u' - u}{t' - t} = \lim \frac{\varphi(s') - \varphi(s)}{t' - t}$

$= \frac{\varphi(s') - \varphi(s)}{s' - s} \; \frac{s' - s}{t' - t}$

$f = \frac{du}{dt} = \frac{d\varphi(s)}{d(s)} \; \frac{ds}{d(t)}$

$\frac{du}{dt} = \frac{du}{ds} \underbrace{\frac{ds}{dt}}_{u}$

$= u \frac{du}{ds} = \frac{d\left(\frac{u^2}{2}\right)}{ds}$

{ r — f α ∧ f δ

$$f = \frac{dv}{dt} = \frac{d\left(\frac{u^2}{2}\right)}{ds}$$

$$\frac{u^2}{2} = \frac{\alpha^2}{2}(a^2 - s^2)$$

$$\frac{d\,\frac{u^2}{2}}{ds} = -\frac{\alpha^2}{2}\,2s = -\alpha^2 s$$

[illegible]

= [illegible]

[[illegible]]

$u^2 = 2fs$

[illegible]

$$\frac{ds}{dt} = \varphi(t)$$

~~$d\,f(t) = f'(t)$~~

$f(t) + a$

$$ds = \varphi(t)\,dt$$

$$\int ds = \int \varphi(t)\,dt$$

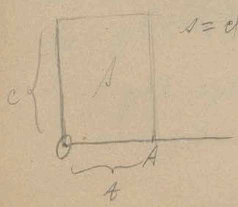
$\int$ u f - - - constante

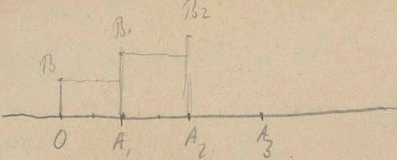
$ds = n\,dt$ { [illegible]

$$\int t^n dt = \frac{t^{n+1}}{n+1}$$

$$\int \frac{dt}{\sqrt{b^2 - t^2}} = \arccos \frac{t}{b}$$

$s = ct$

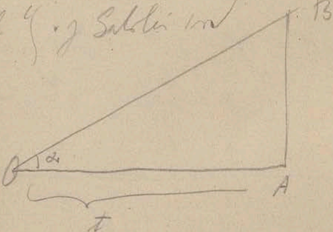






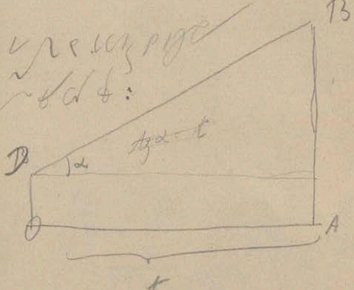
$$\int ds = \int \varphi(t)\,dt$$





$$S = \frac{OA \cdot AB}{2}$$

$$= \frac{t\,gt}{2} = \frac{gt^2}{2}$$



~~$= OA\left(OD + \frac{AB}{2}\right)$~~

$$= ~~t~~ = OA\,OD + \frac{OA\,AB}{2}$$

$$= ct + \frac{gt^2}{2}$$

$$u = OD + gt = c + gt$$

$$\frac{du}{dt} = \varphi(t)$$

$$du = \varphi(t)\,dt$$

$$\int du = \int \varphi(t)\,dt$$

$$u = \int \varphi(t)\,dt$$

$$u = gt + C$$

$$\frac{ds}{dt} = gt + C$$

$$ds = gt\,dt + C\,dt$$

$$s = \int [gt\,dt + c\,dt]$$

$$= g\frac{t^2}{2} + ct + a$$

$$\left.\begin{aligned} \frac{du}{dt} &= g \\ u &= gt + C \\ s &= g\frac{t^2}{2} + ct + a \end{aligned}\right\}$$

$u = \varphi(s)$

$\varphi(s) = \frac{ds}{dt}$

$ds = \varphi(s)\, dt$

$\frac{ds}{\varphi(s)} = dt$

$\chi(s)\, ds = dt$

~~[illegible]~~

$\int \chi(s)\, ds = \int dt$ [illegible]

$u = a\sqrt{s}$

$ds = a\sqrt{s}\, dt$

$\int \frac{ds}{\sqrt{s}} = \int a\, dt$

$2\sqrt{s} = at + b$

$t = \frac{2\sqrt{s} - b}{a}$

$s = \frac{(at+b)^2}{4} = \frac{a^2t^2 + 2abt + b^2}{4}$

[illegible]

[illegible]

[illegible]

$\frac{ds}{dt} = ks$ [illegible]

$\frac{ds}{s} = k\,dt$

$\log s = kt + \alpha$

$s = e^{kt+a} = e^{kt} \cdot e^{a} = e^{kt} A$

$t = 0 \mid s = 0 \mid$

$0 = A$

$s = 0 \cdot e^{kt}$ [illegible]

[illegible]

$\frac{ds}{dt} = kt$

$ds = kt\,dt$

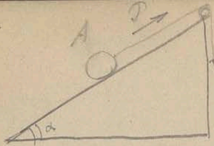
$s = k\frac{t^2}{2} + a$

$t = 0 \mid s = 0 \mid$

$0 = a$

$s = k\frac{t^2}{2}$ [illegible]

[illegible]



[illegible]

[illegible]



[illegible]

[illegible] k [illegible] $\sin\alpha$ [illegible]

$$k = g \sin\alpha$$

$$s = g \sin\alpha \frac{t^2}{2}$$

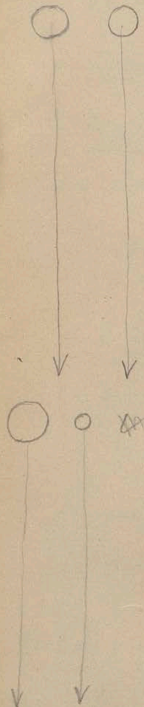
$$\alpha = 90^\circ \quad \left|\ s = g \frac{t^2}{2}\right.$$

$$\alpha = 0^\circ \quad s = 0$$

$$\frac{ds}{dt} = g \sin\alpha \cdot t$$

$$\frac{ds}{dt} = g t$$

$$\frac{dv}{dt} = g \sin\alpha = f$$



$g = \frac{2P}{m}$

$P = \frac{1}{2}\, mg$

$P = mg$

$g = \frac{2s}{t^2}$ [illegible] $g = 31$

[illegible]

[illegible] m [illegible] $g = 31 \cdot 3600$

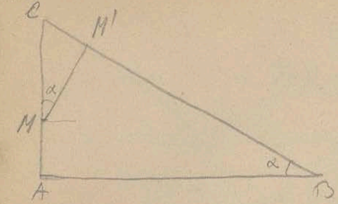
[illegible] 1m [illegible] $g = 9{\cdot}86$

[illegible] 1cm [illegible] $g = 986$

$m = \frac{p}{g}$ [illegible]

[illegible] m [illegible] $1cm^3$ [illegible]

[illegible] 1gr [illegible] $= 986$



$u = gt$

$s = g\frac{t^2}{2}$

$u^2 = g^2t^2 = 2g\frac{gt^2}{2} = 2gs$

II:

$u' = g \sin\alpha\, t$

$s' = g \sin\alpha \frac{t^2}{2}$

$u'^2 = 2g \sin\alpha\, s'$

$s : s' = g : g\sin\alpha$

$= 1 : \sin\alpha$

$s' = s \sin\alpha$

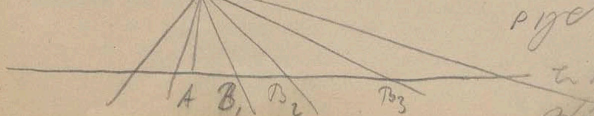
$CM' = CM \sin\alpha$

$s = CA = h$

$s' = CB = l$

$u^2 = 2gh$

$u'^2 = 2gl\sin\alpha = 2gh$



[illegible]

[illegible]

[illegible]

$u_0 = c$

$u_t = c - gt$ | $u' = c - g \sin\alpha\, t$

$s = ct - g\frac{t^2}{2}$ | $s' = ct - g \sin\alpha \frac{t^2}{2}$

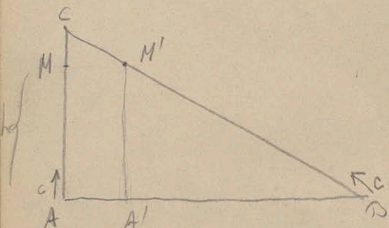
$gt = c - u$

$t = \frac{c-u}{g}$

$u = 0$ [illegible]

$t_1 = \frac{c}{g}$

$h = c\frac{c}{g} - \frac{g}{2}\frac{c^2}{g^2} = \frac{c^2}{2g}$ | $h = \frac{c^2}{2g \sin\alpha}$



$h = \frac{c^2}{2g}$

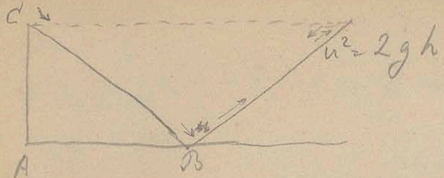
$BM' = \frac{c^2}{2g \sin\alpha}$

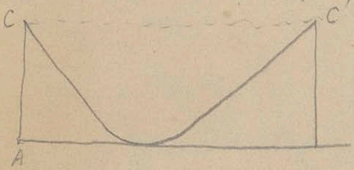
$BM' \sin\alpha = \frac{c^2}{2g}$

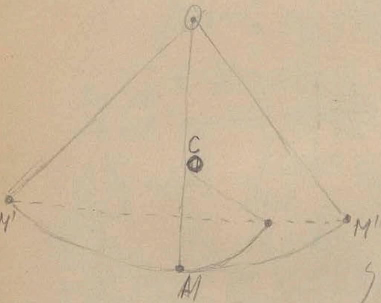
$M'A' = \frac{c^2}{2g} = h$

[illegible] vert. [illegible]

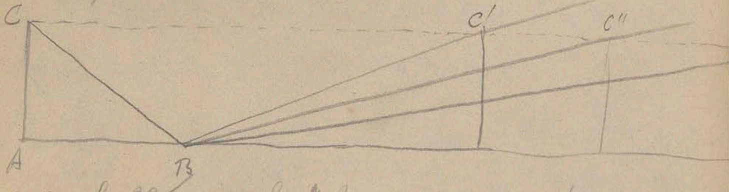
[illegible]







[> 2 CM]



[illegible]

[illegible]

[illegible]

[illegible]

~~$p = mg$~~

$P = Mg$

$P = M \frac{du}{dt} = M \frac{d^2s}{dt^2}$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible]



[illegible]

$$P = M \frac{du}{dt}$$

~~$P\,dt = M\,du$~~

$$M\,du = P\,dt$$

$$\int M\,du = \int P\,dt + C$$

$$t = t_0 \;|\; u = u_0$$

$$M u_0 = P t_0 + C$$

$$M u - M u_0 = P(t - t_0)$$ [[illegible] P const.]

[illegible] P / const.:

$$M u = \int P\,dt + C$$

$$M u_0 = \int P\,dt_0 + C$$

$$M u - M u_0 = \int_{t=t} P\,dt - \int_{t=t_0} P\,dt$$

[illegible]

$$[illegible] = \int_{t_0}^{t} P\,dt$$ [illegible]

[illegible] mult.

[illegible] t_0

$$f = \frac{d\left(\frac{u^2}{2}\right)}{ds} \qquad P = \frac{m\,d\frac{u^2}{2}}{ds} \qquad m\,d\frac{u^2}{2} = p\,ds$$

$$s = s_0 \;|\; u = u_0 \qquad m\frac{u^2}{2} - m\frac{u_0^2}{2} = \int_{s_0}^{s} p\,ds$$

[illegible]:

$$m\frac{u^2}{2} - m\frac{u_0^2}{2} = p(s - s_0)$$

[illegible] $s_0 - s$

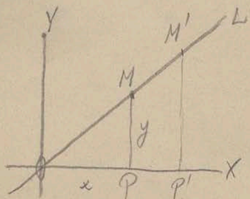
[illegible] [illegible]

[illegible]

[illegible] s_0 [illegible] =

[illegible]

[illegible] [Kinematik]



$OM = s$

$OP = x$

$x = OM \cos\alpha = s\cos\alpha$

$y = MP = s\sin\alpha$

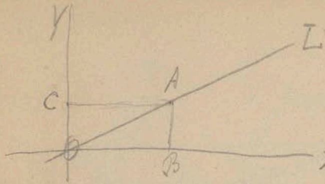
$$u = \frac{ds}{dt}$$

$$\frac{dx}{dt} = \text{[illegible]} = \frac{ds}{dt}\cos\alpha = u\cos\alpha$$

$$\frac{dy}{dt} = \text{" [illegible]} = \frac{ds}{dt}\sin\alpha = u\sin\alpha$$

[illegible]

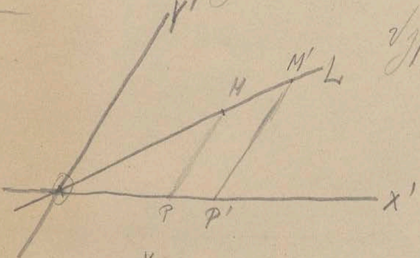
[illegible]

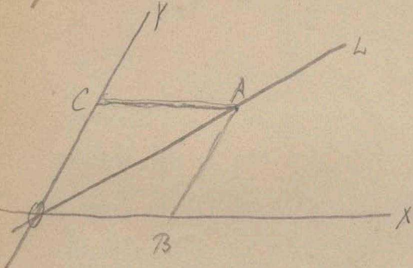


OA = [illegible]

OB = e [illegible] x
AB " " y } Compon[illegible]

[illegible]





[illegible]

$x = s \cos\alpha$

$$\frac{dx}{dt} = \frac{ds}{dt}\cos\alpha = u\cos\alpha$$

$$\frac{d^2x}{dt^2} = \frac{d^2s}{dt^2}\cos\alpha = \frac{du}{dt}\cos\alpha \quad = [illegible]$$

[illegible]

Parallelogram e [illegible] Kinematic[illegible]

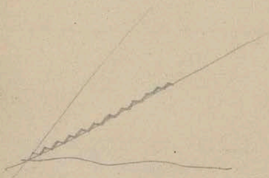
$p = \gamma$.

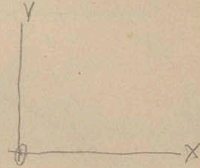
$$m\frac{d^2x}{dt^2} = \underbrace{m\frac{du}{dt}}_{p}\cos\alpha$$

[illegible]

[illegible]

Coordi. [illegible]





$1-\cos\alpha = \sin^2\frac{\alpha}{2}$

$1-\cos\alpha = \frac{\alpha}{2}$ [illegible]

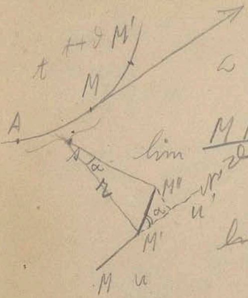
$1-x^2 = 1-\frac{\alpha^2}{2}$

$1-x^2 = 1-\frac{x^2}{2}+\left(\frac{x^2}{2}\right)^2$

$1-2x^2 = 1-2\frac{x^2}{2} \; 2\left(\frac{x^2}{2}\right)^2$

[illegible]

13/11



[illegible]

$\lim \frac{MM'}{\vartheta} = u$ [illegible]

~~$\lim \frac{u'-u}{\vartheta} = $ [illegible]~~

[illegible]

$\lim \frac{u'\cos\alpha - u}{\vartheta} = $ [illegible]

$1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$

$\cos\alpha = 1-2\sin^2\frac{\alpha}{2}$

$= 1 - 2\frac{\alpha^2}{4}$

$= 1 - \frac{\alpha^2}{2}$

$\frac{u'\cos\alpha - u}{\vartheta} = \frac{u'-u}{\vartheta} - \frac{u'\alpha^2}{2\vartheta}$

$\frac{du}{dt}$

$M'M'' = r\alpha$ $\qquad r\alpha = u'\vartheta$

$M'M'' = u'\vartheta$ $\qquad \alpha = \frac{u'\vartheta}{r}$

$r = $ [illegible]

$\lim \frac{u'\cos\alpha - u}{\vartheta} = \frac{u'-u}{\vartheta} - \frac{u'\alpha^2}{2\vartheta} = \frac{du}{dt}$ [scribbled out] $- \frac{u'^3\vartheta^2}{2\vartheta r^2} =$

$= \frac{du}{dt} = $ [illegible] MM' [illegible]

$\lim \frac{u'\sin\alpha}{\vartheta} = $ [illegible] $M'O$

/.

$$\lim \frac{u' \sin\alpha}{\vartheta} = \frac{u'\alpha}{\vartheta} = \frac{u'^2 \vartheta}{r\vartheta} = \frac{u'^2}{r}$$

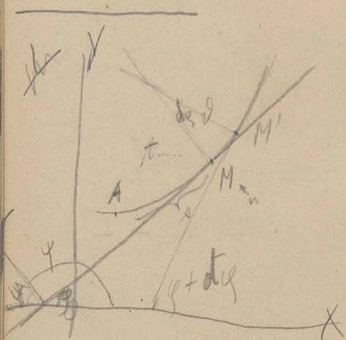
= Centripetalkraft

= [illegible]

[illegible]

[illegible]

Polar Coord.



$$u = \frac{ds}{dt}$$

$$\frac{dx}{dt} = u \cos\varphi$$

$$\frac{dy}{dt} = u \sin\varphi$$

$$\frac{d^2x}{dt^2} = \frac{du}{dt}\cos\varphi - u \sin\varphi \frac{d\varphi}{dt}$$

$$\frac{d^2y}{dt^2} = \frac{du}{dt}\sin\varphi + u\cos\varphi \frac{d\varphi}{dt}$$

component [illegible]

[illegible]

$u\frac{d\varphi}{dt}$ $\quad \psi$

$$\frac{u\,d\varphi}{dt}\cos\psi = -\frac{u\,d\varphi}{dt}\sin\varphi$$

$$\frac{u\,d\varphi}{dt}\sin\psi = u\cos\varphi\frac{d\varphi}{dt}$$

$\cos\psi = -\sin\varphi$

$\sin\psi = \cos\varphi$

$\psi = 90 + \varphi$

$\cos\psi = -\sin\varphi$

$\sin\psi = \cos\varphi$ [illegible]

[illegible]

$$r\,d\varphi = ds$$

$$r = \frac{ds}{d\varphi}$$

$$d\varphi = \frac{ds}{r}$$

$$\frac{u\,d\varphi}{dt} = \frac{u\,ds}{r\,dt}$$

$$= \frac{u}{r}\cdot\frac{ds}{dt} = \frac{u^2}{r}$$

[illegible]

$$\frac{dx}{dt} = u\cos\varphi = u\frac{dx}{ds}$$

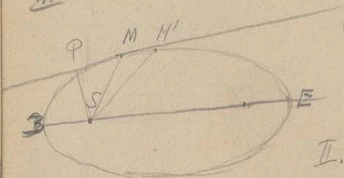
[illegible]

14/4 Entwicklung d. Gravitationsg. aus d. Kepplerschen G. in rechtw. Coordinaten

I Kepler 1 — Ell [illegible]

II ~

III ~~



$$MM'S = \frac{MM'.SP}{2}$$

$$= \frac{ds\, p}{2}$$

II. $\frac{ds\, p}{2\vartheta}$ = constante [illegible]

$$\frac{ds\, p}{2\vartheta} = C$$

$$\lim \frac{MM'}{\vartheta} = u$$

$$\frac{u p}{2} = C$$

$$u = \frac{2c}{p} \qquad c = \frac{\pi a b}{T}$$

[illegible] Perih [illegible] Aphel

$$\frac{du}{dt} = -\frac{2c}{p^2}\frac{dp}{dt} = \text{[illegible]}$$

[illegible] B S E [illegible]

$$f = \frac{u^2}{\rho} \qquad C: \; u = \frac{2c}{BS} \qquad f = \frac{4c^2}{\rho\,(BS)^2}$$

, E:

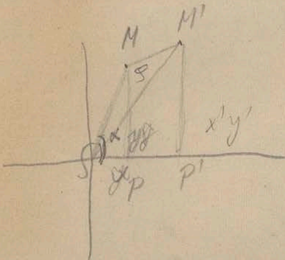
$n = \frac{2c}{ES}$ $\quad f' = \frac{4c^2}{\rho'(ES)^2} = \frac{4c^2}{\rho(ES)^2}$

$\rho = \rho'$

$f : f' = \frac{1}{BS^2} : \frac{1}{ES^2}$

[illegible]

[illegible]



$MM'S = SMP + MPP'M' - SM'P'$

$f =$

$= \frac{xy}{2} + \frac{(x'-x)(y'+y)}{2} - \frac{x'y'}{2} =$

$= \frac{xy}{2} + \frac{x'y}{2} - \frac{x'y'}{2} - \frac{xy'}{2} + \frac{x'y'}{2} - \frac{xy}{2} =$

$= \frac{x'y - xy'}{2}$

$x' = x + \frac{dx}{dt}\vartheta \qquad y' = y + \frac{dy}{dt}\vartheta$ | $DSM = F$

$MM'S = \frac{y\frac{dx}{dt}\vartheta - x\frac{dy}{dt}\vartheta}{2}$

$\frac{\varphi}{\vartheta} = \frac{1}{2}\left[y\frac{dx}{dt} - x\frac{dy}{dt}\right]$

DSM = F

[illegible] $\frac{e}{\vartheta} = \frac{dF}{dt} = $ ~~[illegible]~~ $= C$

$$y\frac{dx}{dt} - x\frac{dy}{dt} = 2c$$

$$y\frac{d^2x}{dt^2} + \frac{dy}{dt}\frac{dx}{dt} - x\frac{d^2y}{dt^2} - \frac{dx}{dt}\frac{dy}{dt} = 0$$

$$y\frac{d^2x}{dt^2} - x\frac{d^2y}{dt^2} = 0$$

[illegible]

$$\frac{d^2x}{dt^2} = l x \qquad \frac{d^2y}{dt^2} = l y$$

$$\frac{d^2x}{dt^2} = l r \frac{x}{r} = l r \cos\alpha \qquad \frac{d^2y}{dt^2} = l r \sin\alpha$$

[illegible]

[illegible]

[illegible]

---

consq. of Kepler law [illegible]

Centralbewegung [illegible] const [illegible]]

Gx l § d:

d s I I

$$\text{I} \quad \frac{(x-e)^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{II} \quad y\frac{dx}{dt} - x\frac{dy}{dt} = 2c \qquad \Big| \frac{y}{b^2} \qquad \frac{y}{b^2}$$

$$\frac{(x-e)}{a^2}\frac{dx}{dt} + \frac{y}{b^2}\frac{dy}{dt} = 0 \qquad \Big| x \qquad x$$

$$\left(\frac{y^2}{b^2} + \frac{(x-e)}{a^2}x\right)\frac{dx}{dt} = 2c\frac{y}{b^2}$$

$$\Big[\underbrace{\frac{y^2}{b^2} + \frac{(x-e)^2}{a^2}}_{1} + e\frac{(x-e)}{a^2}\Big] \text{ - - - - - - -}$$

$$\left[1 + \frac{e(x-e)}{a^2}\right]\frac{dx}{dt} = \frac{2cy}{b^2}$$

$$r^2 = x^2 + y^2$$

$$= x^2 + b^2 - \frac{b^2}{a^2}(x-e)^2$$

$$= \frac{a^2x^2 + a^2b^2 - b^2x^2 + 2b^2ex - b^2e^2}{a^2}$$

$$= \frac{e^2x^2 + b^4 + 2b^2ex}{a^2}$$

$$r^2 = \left(\frac{b^2+ex}{a}\right)^2 = \frac{(b^2+ex)^2}{a^2}$$

$$r = \frac{b^2+ex}{a}$$

$eR = ar - b^2$

$$\left[1 + \frac{ar - b^2 - e^2}{a^2}\right] = 1 + \frac{ar - ar}{a^2} = \frac{ar}{a^2} =$$

$$\frac{~~ar~~}{a}\frac{dx}{dt} = \frac{2cy}{b^2}$$

18/11

$$\frac{1}{a}\frac{dx}{dt} = \frac{2cy}{b^2}$$

$$\frac{dx}{dt} = \frac{2ac}{b^2}\frac{y}{r}$$

$$\frac{d^2x}{dt^2} = \frac{2ac}{b^2}\left[\frac{1}{r}\frac{dy}{dt} - \frac{y}{r^2}\frac{dr}{dt}\right]$$

$$= \frac{2ac}{b^2r^3}\left[r^2\frac{dy}{dt} - yr\frac{dr}{dt}\right]$$

$$r^2 = x^2 + y^2$$

$$r\frac{dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

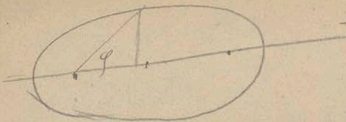
$$\frac{d^2x}{dt^2} = \frac{2ac}{b^2r^3}\left[\frac{x^2dy}{dt} + y^2\frac{dy}{dt}\right.$$

$$\left. - xy\frac{dx}{dt} - y^2\frac{dy}{dt}\right]$$

$$= -\frac{2acx}{b^2r^3}\left[y\frac{dx}{dt} - x\frac{dy}{dt}\right]$$

$$\frac{d^2x}{dt^2} = -\frac{4ac^2x}{r^3b^2}$$

$$= -\frac{4ac^2}{b^2r^2}\frac{x}{r}$$



$\frac{x}{r} = \cos\varphi$

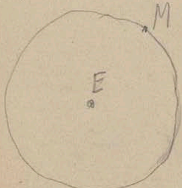
$$f = -\frac{4ac^2}{b^2r^2} = -\frac{4\pi^2a^3b^2}{b^2T^2r^2} = -\frac{4\pi^2a^3}{T^2}\frac{1}{r^2}$$

$\pi ab = cT$

$a^3 : a'^3 = T^2 : T'^2$

$$\frac{a^3}{T^2} = \frac{a'^3}{T'^2} = - - -$$

[illegible]



[illegible]

$g : g' = \frac{1}{E^2} : \frac{1}{\mu^2}$

$$g' = \frac{gE^2}{\mu^2}$$

$\frac{u^2}{\mu} = g'$ $\qquad \frac{u^2}{\mu} = g\frac{E^2}{\mu^2}$

$\frac{2\pi\mu}{T\mu} = n$

Gravit.

$f = \frac{k m_1 m_2}{r^2}$

$\frac{4\pi^2\mu^2}{T^2\mu} = g\frac{\varepsilon^2}{\mu^2}$

$$T^2 = \frac{4\pi^2\mu^2}{\mu}\frac{\mu^2}{g\varepsilon^2}$$

$$T^2 = \frac{4\pi^2\mu^3}{g\varepsilon^2}$$

$$T = \frac{2\pi\mu}{\varepsilon}\sqrt{\frac{\mu}{g}}$$

$\mu = 60{\cdot}27\,\varepsilon$

$$T = 2\pi\, 60{\cdot}27\sqrt{\frac{60{\cdot}27\,\varepsilon}{g}}$$

$g = 9{\cdot}806$

$\frac{\pi\varepsilon}{2} = 10^7$

$$T = 2.\,60{\cdot}27\sqrt{\frac{60{\cdot}27 . 2 . 10^7 . \pi}{9{\cdot}806}}$$

$\varepsilon = \frac{2.10^7}{\pi}$

$$T = 2.60270\sqrt{\frac{602{\cdot}7 . 2\pi}{9{\cdot}806}}$$

2·780101	0991492	2369000''
0·301030	586809	= 39486'
0·497150		= 658h
3·578281	1·293940	= 27·4 / 345
	0·301030	
	4·780101	

20/11 [illegible] Ell. [illegible] R Vector [illegible]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y\frac{dx}{dt} - x\frac{dy}{dt} = 2c \quad \Big| \; \frac{y}{b^2} \quad \Big| \; -\frac{x}{a^2}$$

$$\frac{2x}{a^2}\frac{dx}{dt} + \frac{2y}{b^2}\frac{dy}{dt} = 0 \quad \Big| \; x \quad \Big| \; y$$

$$\frac{x^2}{a^2}\frac{dx}{dt} + \frac{y^2}{b^2}\frac{dx}{dt} = \frac{2cy}{b^2}$$

$$\frac{dx}{dt}\left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right] = \frac{2cy}{b^2}$$

$$\frac{dx}{dt} = \frac{2cy}{b^2} \qquad \frac{dy}{dt} = -\frac{2cx}{a^2}$$

$$\frac{d^2x}{dt^2} = \frac{2c}{b^2}\frac{dy}{dt} = -\frac{4c^2x}{a^2b^2} \qquad \frac{d^2y}{dt^2} = -\frac{2c}{a^2}\frac{dx}{dt} = -\frac{4c^2y}{a^2b^2}$$

$$f = -\frac{4c^2r}{a^2b^2}$$ [illegible] prop. [illegible]

[illegible] Ell. [illegible] elast. [illegible]

[illegible]:

$$\frac{d^2x}{dt^2} = -\frac{4c^2}{a^2b^2}x$$ Charakteristik [illegible]

[illegible] Ellipse [illegible] RV. [illegible]

$$\frac{d^2x}{dt^2} = -\alpha^2 x$$

$x = A\sin\alpha t \quad x = B\cos\alpha t$ } [illegible]

$x = A\sin\alpha t + B\cos\alpha t$ [illegible]

$y = C\sin\alpha t + D\cos\alpha t$ | [illegible]

[illegible] Ampl. [illegible] $B = a$

$\frac{dx}{dt} = 0 \; | \; A = 0 \qquad A = 0$

$x = a\cos\alpha t \qquad | \; A = 0 \quad y = 0$

$y = C\sin\alpha t \qquad y = D = 0 \qquad y = 0$

$\;\; = b\sin\alpha t$

[illegible]:

$\frac{x}{a} = \cos\alpha t$ $\qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Big\}$ [illegible]

$\frac{y}{b} = \sin\alpha t$

[illegible]:

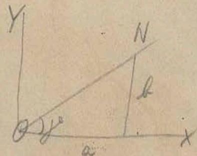
$y = b \sin\alpha t$

$x = a \sin\alpha t$

$\frac{y}{x} = \frac{b}{a}$

$y = \frac{b}{a} x$

[illegible]



$y = b \sin\alpha t$

$x = a \sin\alpha(t - \vartheta) \quad \Big| \quad t = \vartheta, \; x = 0$

$x = a \sin\alpha t \cos\alpha\vartheta - a \sin\alpha\vartheta \cos\alpha t$

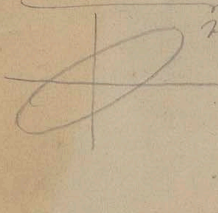
$\frac{x}{a} = \frac{y}{b} \cos\alpha\vartheta - \sin\vartheta \cos\alpha t$

$\left(\frac{x}{a} - \frac{y}{b}\cos\alpha\vartheta\right)^2 = \cos^2\alpha t \sin^2\alpha\vartheta = \left(1 - \frac{y^2}{b^2}\right)\sin^2\alpha\vartheta$

$\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\alpha\vartheta + \frac{y^2}{b^2}\cos^2\alpha\vartheta = \sin^2\alpha\vartheta - \frac{y^2}{b^2}\sin^2\alpha\vartheta$

$= \frac{y^2}{b^2}$

$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha\vartheta + \frac{y^2}{b^2} = \sin^2\alpha\vartheta$



[illegible] Curve [illegible] Ellipse

[illegible] $\vartheta = 0$ [illegible]

[illegible] $\alpha\vartheta = \frac{\pi}{2}$ [illegible] Elli[illegible]

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

[illegible]

cond. [illegible] 87]

$F(x, y) = 0$

Interferenz [illegible] Spectrallin[illegible]

[illegible]

[illegible] spectra [illegible]

H_2O HCl SO_2 CO_2

Spectra [illegible]

Spectral [illegible]

[illegible]	[illegible]	0,09
Na	Pb	1000
Na	Ba	1005
Te	N	1366
Os	Ar	1737
Cr	N	2326

[illegible]

[illegible] spectra [illegible]

$2B$, Cl_3P $CuCl_2$ CuI_2

absorptions [illegible] [[illegible]]

[illegible] Cr_2O [illegible] spectra

[illegible] Blau[illegible] [illegible] spectr[illegible]

[illegible] scala

[illegible] pyrometer

$\ell = 3433 \frac{l}{g}$

$l = 7357\cdot4$

$g = \frac{d}{s^2 l}$

46·4915

46·4915 : x = 90 : 10

x = 464·915 : 90

= 5·16557 = g

3443

42

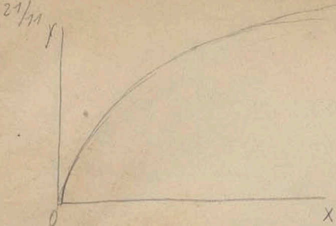
Dr. Josef Stefan

II.

Mechanik

I.S. 90/91 M Smoluchowski

21/11



$$\frac{d^2x}{dt^2} = g$$

$$\frac{d^2y}{dt^2} = 0$$

$$\frac{dy}{dt} = c$$

$$y = ct \quad \to \int \Big|_{y_0=0}^{t=0}$$

$$\frac{dx}{dt} = gt + a$$

$$x = g\frac{t^2}{2} + \underset{=0}{a}t + \underset{=0}{b} \quad \to \int \Big|_{\frac{dx}{dt}=0}^{t=0} \Big\} a=0$$

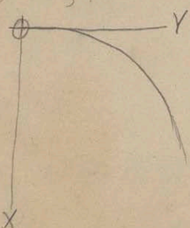
$$x = g\frac{t^2}{2}$$

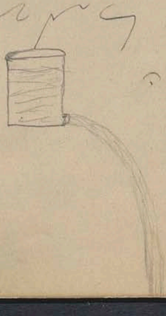
$$\underline{y = ct}$$

$$\frac{x}{y^2} = \frac{g}{2c^2}$$

$$y^2 = \frac{2c^2}{g}x \quad = \text{Parabel } [illegible] \; p = \frac{c^2}{g}$$

[illegible] [Galilei]





[illegible]

X Axe [illegible]

[illegible] Y Axe

[illegible]

[illegible]

[illegible] Eigenm.

[illegible]

[illegible]

[illegible]

[illegible] Wilh. Weber Halle elekt. [illegible]

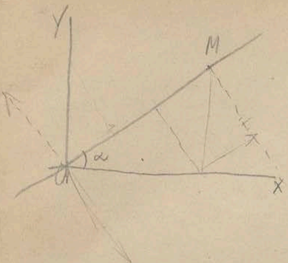
[illegible]

[illegible] Newton [illegible]

[illegible]

[illegible]

[illegible]



$$\frac{d^2s}{dt^2} =$$

$$x = s\cos\alpha$$

$$\frac{d^2x}{dt^2} = \frac{d^2s}{dt^2}\cos\alpha$$

$$\frac{d^2y}{dt^2} = \frac{d^2s}{dt^2}\sin\alpha$$

[illegible]

$$\frac{d^2x}{dt^2}\cos\alpha$$

$$\frac{d^2y}{dt^2}\sin\alpha$$

$$\frac{d^2s}{dt^2}\cos^2\alpha + \frac{d^2s}{dt^2}\sin^2\alpha = \frac{d^2s}{dt^2}$$

[illegible]

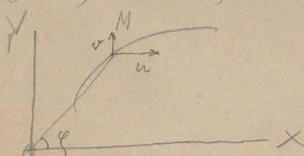
$$\frac{d^2x}{dt^2}\sin\alpha$$

$$\frac{d^2y}{dt^2}\cos\alpha$$

$$\frac{d^2x}{dt^2}\sin\alpha - \frac{d^2y}{dt^2}\cos\alpha$$

$$\frac{d^2s}{} \quad = 0$$

[illegible]



$$OM = r$$

$$x = r\cos\varphi$$

$$y = r\sin\varphi$$

$$\frac{dx}{dt} = \frac{dr}{dt}\cos\varphi - r\sin\varphi\frac{d\varphi}{dt}$$

$$\frac{dy}{dt} = \frac{dr}{dt}\sin\varphi + r\cos\varphi\frac{d\varphi}{dt}$$

$u \cos\varphi + v \sin\varphi$

~~[illegible]~~

$= \frac{dx}{dt}\cos\varphi + \frac{dy}{dt}\sin\varphi \qquad = \frac{dr}{dt}$

Ganz:

$$\frac{d^2x}{dt^2} = \frac{d^2r}{dt^2}\cos\varphi - 2\frac{dr}{dt}\sin\varphi\frac{d\varphi}{dt} - \frac{dr}{dt}\sin\varphi -$$

$$- r\cos\varphi\left(\frac{d\varphi}{dt}\right)^2 -$$

$$- r\sin\varphi\frac{d^2\varphi}{dt^2}$$

$$\frac{d^2y}{dt^2} = \frac{d^2r}{dt^2}\sin\varphi + 2\frac{dr}{dt}\cos\varphi\frac{d\varphi}{dt} - r\sin\varphi\left(\frac{d\varphi}{dt}\right)^2 +$$

$$+ r\cos\varphi\frac{d^2\varphi}{dt^2}$$

$$\frac{d^2x}{dt^2}\cos\varphi + \frac{d^2y}{dt^2}\sin\varphi =$$

$$= \frac{d^2r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2 \qquad \left[\ [illegible]\ \frac{d^2r}{dt^2}\,!\ \right]$$

[illegible]

II Yng ⊥ z RV

$$\frac{d^2y}{dt^2}\cos\varphi - \frac{d^2x}{dt^2}\sin\varphi = \sigma$$

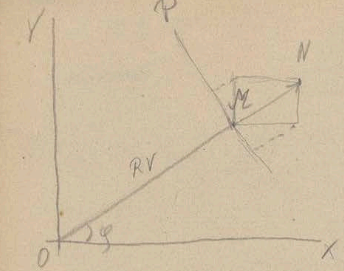
$$= 2\frac{dr}{dt}\frac{d\varphi}{dt} + r\frac{d^2\varphi}{dt^2}$$ = Yng ⊥ z RV

$$= \frac{1}{r}\left[2r\frac{dr}{dt}\frac{d\varphi}{dt} \quad r^2\frac{d^2\varphi}{dt^2}\right]$$

$$= \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\varphi}{dt}\right)$$

Yng i Polarcoordinat

25/11



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$\frac{dx}{dt}\cos\varphi + \frac{dy}{dt}\sin\varphi$$

$$\frac{x}{r}\frac{dx}{dt} + \frac{y}{r}\frac{dy}{dt} = MK = \frac{dr}{dt}$$ [illegible]

$$r^2 = x^2 + y^2$$

$$\frac{r\,dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$v(MP) = S$$

$$S = \frac{dy}{dt}\cos\varphi - \frac{dx}{dt}\sin\varphi = r\frac{d\varphi}{dt}$$

$$\frac{dx}{dt} = \frac{dr}{dt}\cos\varphi - r\sin\varphi\frac{d\varphi}{dt}$$

$$\frac{dy}{dt} = \frac{dr}{dt}\sin\varphi + r\cos\varphi\frac{d\varphi}{dt}$$

[illegible] $r\,d\varphi$ [illegible] dr [illegible]

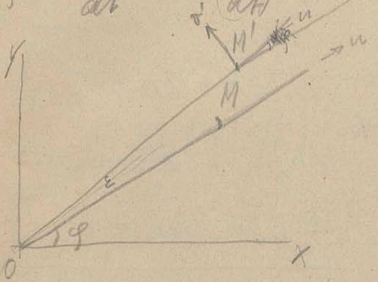
$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = \left(\frac{ds}{dt}\right)^2 = \text{[illegible]}$$

$$ds^2 = dr^2 + r^2 d\varphi^2$$ [illegible]

Polarcoor[illegible]

yerz $\perp R V = \rho$

$$\rho = \frac{d^2r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2$$



[comp. v. de φ sc. de RV]

$$u = \frac{dr}{dt}$$

$$\frac{u'\cos\varepsilon - u - v'\sin\varepsilon}{\tau} = \rho$$

$$u' = u + \frac{du}{dt}\tau$$

$$= \frac{dr}{dt} + \frac{d^2r}{dt^2}\tau$$

$$v = \frac{r\,d\varphi}{dt}$$

$$v' = \left(v + \frac{dv}{dt}\tau\right)$$

$$\rho = \frac{\left(u + \frac{du}{dt}\tau\right)\left(1 - \frac{\varepsilon^2}{2}\right) - u - \left(v + \frac{dv}{dt}\tau\right)\varepsilon}{\tau}$$

$$\frac{r\varepsilon}{\tau} = v \perp RV$$

$$\varepsilon = \frac{v\tau}{r}$$

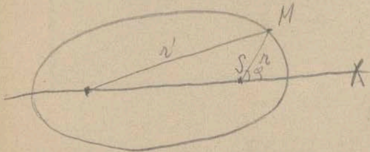
$$\frac{u + \frac{du}{dt}\tau - (\ )\frac{v^2\tau^2}{2} - u - \left(v + \frac{dv}{dt}\tau\right)\frac{v\tau}{r}}{\tau}$$

$$\rho = \lim \frac{du}{dt} - \frac{v^2}{r} - [\ \ldots\ ]\tau$$

$$\rho = \frac{du}{dt} - \frac{v^2}{r} = \frac{d^2r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2$$

[illegible]

[illegible]; Ableitung d. Gravitationsg. a. Keplers S. (Polarcoord.)



$$r' + r = 2a$$

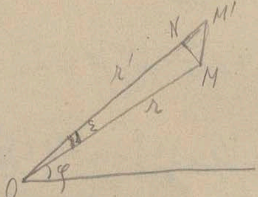
$$r'^2 = r^2 + 4e^2 + 4er\cos\varphi$$

$$r'^2 = 4a^2 - 4ar + r^2$$

$$4a^2 - 4ar + r^2 = 4e^2 + 4er\cos\varphi$$

$$a^2 - e^2 = r(a + e\cos\varphi)$$

IKG $\left\{ \quad r = \frac{b^2}{a + e\cos\varphi} \right.$



$$MN = r\varepsilon$$

$$\frac{r' r \varepsilon}{2}$$

$$= c = \frac{r' r\, \varepsilon}{2\tau} = \frac{r' r}{2}\frac{d\varphi}{dt}$$

$$\lim \frac{\varepsilon}{t} = \frac{d\varphi}{dt} \qquad = \lim\left[r + \frac{dr}{dt}\tau\right]\frac{r}{2}\frac{d\varphi}{dt}$$

$$r' = r + \frac{dr}{dt}\tau \qquad = \frac{r^2}{2}\frac{d\varphi}{dt} = c$$

II K G $r^2 \frac{d\varphi}{dt} = 2c$

[illegible]

$$\rho = \frac{d^2r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2$$

$$\sigma = \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\varphi}{dt}\right)$$

$$\left[r^2\frac{d\varphi}{dt} = 2c\right]$$

$$\sigma = 0$$

[illegible]

$$\frac{dr}{dt} = +\frac{b^2 e\sin\varphi \frac{d\varphi}{dt}}{(a+e\cos\varphi)^2} = \frac{b^2 e\sin\varphi}{(a+e\cos\varphi)^2}\frac{d\varphi}{dt}$$

$$= \frac{r^2}{b^2} e\sin\varphi\frac{d\varphi}{dt}$$

$$= \frac{2ce}{b^2}\sin\varphi$$

$$\frac{d^2r}{dt^2} = \frac{2ce}{b^2}\cos\varphi\frac{d\varphi}{dt}$$

$$\rho = \frac{2ce}{b^2}\cos\varphi\frac{d\varphi}{dt} - r\frac{2c}{r^2}\frac{d\varphi}{dt}$$

$$= 2c\frac{d\varphi}{dt}\left[\frac{e\cos\varphi}{b^2} - \frac{a+e\cos\varphi}{b^2}\right] = -\frac{2ac}{b^2}\frac{d\varphi}{dt}$$

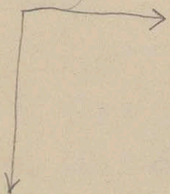
$$= -\frac{2ac}{b^2}\frac{2c}{r^2} = -\frac{4ac^2}{b^2}\frac{1}{r^2}$$

26/11

[illegible]

[illegible]

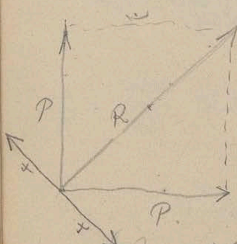
[illegible]



[illegible]

[illegible]

[illegible]



[illegible]

[illegible]

$R = kP$ [illegible]

2 [illegible]

R [illegible] P P [illegible] R [illegible] P P S x x

[illegible] P P [illegible] R P P R x x

[illegible] $x = \frac{R}{2}$ [illegible] $x + \frac{R}{2}$ $x + \frac{R}{2}$ [illegible] Res?

$P = kx = k\frac{R}{2}$ [illegible]

[illegible]

$R = kP$

$P = k\frac{R}{2}$

$PR = \frac{k^2}{2} PR$

$\frac{k^2}{2} = 1$

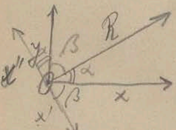
$k = \sqrt{2}$ $R = P\sqrt{2}$

$R^2 = P^2 \cdot 2 = P^2 + P^2$

[illegible]

[illegible]:

[illegible]



$x = R\lambda$

$x = R f(\alpha)$

$y = R f(\beta)$

$x' = x''$

$R = R' + R''$ [illegible] $R' R'' x' x'' P x y$

Analuse $\begin{cases} x' + R' = x \\ R'' + x'' = y \end{cases}$

$x' = x f(\beta)$ $R' = x f(\alpha)$

$x'' = y f(\alpha)$ $R'' = y f(\beta)$

[illegible]

$x' = x''$

$x f(\beta) = y f(\alpha)$

$x f(\alpha) + y f(\beta) = R$

$x = R f(\alpha)$

$y = R f(\beta)$

$x f(\alpha) + y f(\beta) = R\left[f(\alpha)^2 + f(\beta)^2\right]$

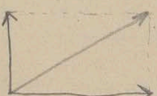
$R = R\left[f(\alpha)^2 + f(\beta)^2\right]$

$f(\alpha)^2 + f(\beta)^2 = 1$

$x^2 + y^2 = R^2\left[f(\alpha)^2 + f(\beta)^2\right]$

$x^2 + y^2 = R^2$

[illegible] Resul… [illegible]



$f(\alpha) = \frac{x}{R}$ = ~~[illegible]~~

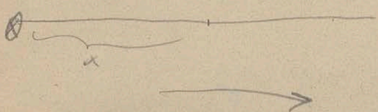
$= \cos\alpha$

$f(\beta) = \cos\beta$

[illegible]

[illegible]

[illegible]



masse = m

Kraft = X

$$m \frac{d^2x}{dt^2} = X$$

$$X = mg$$

$$m \frac{d^2x}{dt^2} = mg$$

$$\frac{d^2x}{dt^2} = g$$

$$\frac{dx}{dt} = gt + C$$

$$x = g\frac{t^2}{2} + Ct + A$$

A. $t = 0$ | $\frac{dx}{dt} = 0$ | $x = 0$

$$x = g\frac{t^2}{2}$$

[illegible]

[illegible]

mg ~~[illegible]~~

$-\mu g$

$-\alpha f(u)$

$$m\frac{d^2x}{dt^2} = mg - \mu g - \alpha f(u)$$

[illegible]

$$m\frac{du}{dt} = mg - \mu g - \alpha f(u)$$

$$\frac{du}{dt} = g - \frac{\mu}{m} g - \frac{\alpha}{m} f(u)$$

$$= g\left[1 - \frac{\mu}{m}\right] - \frac{\alpha}{m} f(u)$$

$$\frac{du}{g\left[1-\frac{\mu}{m}\right] - \frac{\alpha}{m} f(u)} = dt$$

27/11

[illegible]

[illegible]

[illegible]

$f(u) = u$

$$\frac{du}{g\left[1-\frac{\mu}{m}\right] - \frac{\alpha}{m}u} = dt$$

$$\underbrace{g\left[1-\frac{\mu}{m}\right]}_{g'} \qquad \frac{\alpha}{mg'} = b$$

$$\frac{du}{g' - bg'u} = dt$$

$$\frac{du}{1-bu} = g'\,dt$$

$$\frac{b\,du}{\quad} = bg'\,dt$$

$$\log[1-bu] = -bg't + C$$

$$1-bu = e^{-bg't+C} \qquad e^{C} = A$$

$$= Ae^{-bg't}$$

$$t=0 \,|\, u=0 \qquad 1 = A \qquad 1-bu = e^{-bg't}$$

$$u = \frac{1 - e^{-bg't}}{b}$$

[illegible]

$$u = \frac{1}{b} \qquad \Big| t = \infty$$

[illegible]

[illegible]

[illegible] $b = 0$

$$u = \frac{0}{0}$$

$$= \frac{1 - \left[1 - bg't + \frac{b^2 g'^2 t^2}{2} - \frac{b^3 g'^3 t^3}{6} - - - \right]}{b}$$

$$= \frac{bg't - \frac{b^2 g'^2 t^2}{2} ---}{b}$$

$$= g't - \frac{b g'^2 t^2}{2} + ---$$

$|b=0$

$$u = g't$$

$s = ?$

$$u = \frac{dx}{dt} = \frac{1 - e^{-bg't}}{b}$$

$$dx = \frac{dt}{b} - \frac{dt\, e^{-bg't}}{b}$$

$$x = \frac{t}{b} + \frac{e^{-bg't}}{b^2 g'} + B$$

$t = 0 \mid x = 0$

$$0 = \frac{1}{b^2 g'} + B$$

$$x = \frac{t}{b} - \frac{1}{b^2 g'}\left[1 - e^{-bg't}\right]$$

$$x = \frac{t}{b} - \frac{1}{b^2 g'}\left[1 - 1 + bg't - \frac{b^2 g'^2 t^2}{2} + \frac{b^3 g'^3 t^3}{6} \cdots\cdots\right]$$

$$x = \frac{t}{b} - \frac{t}{b} + \frac{g't^2}{2} - \frac{b g'^2 t^3}{6} \cdots\cdots$$

$b = 0$

$$x = \frac{g't^2}{2}$$

| ω [illegible]

ve

$x = \frac{t}{b} \mid t = \infty$

$f(u) = \alpha u^2$

$$\frac{du}{dt} = g' - \frac{\alpha}{m} u^2$$

[[illegible] $= g$

[illegible] $g' = \frac{\alpha}{m}$ [illegible]

[illegible] $= 0$

$$\frac{du}{\quad} = \frac{\quad}{\quad} \qquad \frac{\alpha}{mg'} = b^2$$

$$\frac{du}{1 - b^2u^2} = g'dt$$

$$\frac{du}{2}\left[\frac{1}{1-bu} + \frac{1}{1+bu}\right] = g'dt$$

$$\frac{b\,du}{1-bu} + \frac{b\,du}{1+bu} = 2bg'dt$$

$$-\log(1-bu) + \log(1+bu) = 2bg't + C$$

$$\log\frac{1+bu}{1-bu} = 2bg't + C$$

$t=0 \,|\, u=0 \qquad 0 = C$

$$\frac{1+bu}{1-bu} = e^{2bg't}$$

$$1+bu = e^{\cdots} - bu\, e^{\cdots}$$

$$u = \frac{e^{2bg't}-1}{e^{2bg't}+1} \cdot \frac{1}{b}$$

$$t = \infty \quad \Big| \quad u = \frac{1}{b}$$

$$\frac{dx}{dt} = \frac{1}{b} \frac{e^{2bg't}-1}{e^{2bg't}+1}\, dt$$

$$= \frac{1}{b} \cdot \frac{e^{bg't}-e^{-bg't}}{e^{bg't}+e^{-bg't}}\, dt$$

$$e^{bg't}+e^{-bg't} = Z$$

$$\frac{dZ}{dt} = bg'\left[e^{bg't}-e^{-bg't}\right]$$

$$dx = \frac{1}{b^2 g'} \frac{dZ}{Z}$$

$$x = \frac{1}{b^2 g'} \log\left[e^{bg't}+e^{-bg't}\right] + D$$

$t=0 \mid x=0$

$$0 = \frac{1}{b^2 g'} \log 2 + D$$

$$x = \frac{1}{b^2 g'} \log \frac{e^{bg't} + e^{-bg't}}{2}$$

$$\boxed{b = 0 \mid x = \frac{g't^2}{2}}$$

$$= \frac{1}{b^2 g'} \log\left[1 + \frac{b^2 g'^2 t^2}{2} \text{ --- ---}\right.$$

$$= \frac{1}{b^2 g'} \cdot \left[\frac{b^2 g'^2 t^2}{2} \text{ --- ---}\right.$$

$$= \frac{g' t^2}{2}$$

28/11 [illegible]



$$m\frac{d^2x}{dt^2} = 0$$

$$m\frac{d^2y}{dt^2} = -mg$$

$$\frac{d^2x}{dt^2} = 0$$

$$\frac{d^2y}{dt^2} = -g$$

$$\frac{dx}{dt} = a$$

$$x = at + A$$

$$\frac{dy}{dt} = -gt + b$$

$$y = -g\frac{t^2}{2} + bt + B$$

$t=0 \mid x=0 \mid y=0$

$A=0$

$B=0$

$t=0 \quad \frac{dx}{dt} = a \quad \frac{dy}{dt} = b$

[illegible]

[illegible] negat.

$$\frac{dy}{dt} = 0 = -gt + b$$

$$t = \frac{b}{g}$$

$$x = at \qquad x = \frac{ab}{g}$$

$$y = -b\frac{t^2}{2} + bt$$

$$y = -\frac{b^2}{2g} + \frac{b^2}{g}$$

$$h = \frac{b^2}{2g}$$

$y = 0$

$bt - g\frac{t^2}{2} = 0 \qquad \Big| \; t = 0$

$b = g\frac{t}{2}$

$t = \frac{2b}{g}$ [illegible]

$x = \frac{2ab}{g}$ = [illegible] $= w$

$h = \frac{b^2}{2g}$

16 [illegible]:

$y = \frac{bx}{a} - \frac{g}{2}\frac{x^2}{a^2}$ = [illegible]

y [illegible]

[illegible] 0 − [illegible] x [illegible]

[illegible] − [illegible] c [illegible]

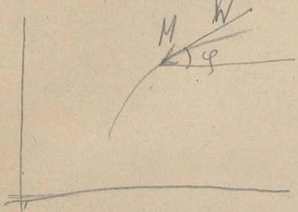
$a = c \cos\alpha$

$b = c \sin\alpha$

$w = \frac{2ab}{g} = \frac{2c^2 \sin\alpha\cos\alpha}{g} = \frac{c^2 \sin 2\alpha}{g}$

[illegible] $\measuredangle\alpha = 45^0$

[illegible]:



$$m\frac{d^2x}{dt^2} = W\cos\varphi$$

$$m\frac{d^2y}{dt^2} = -mg + W\sin\varphi$$

$$W = -\alpha f(u)$$

I. $f(u) = u$

$$m\frac{d^2x}{dt^2} = -\alpha u\cos\varphi$$

$$m\frac{d^2y}{dt^2} = -mg - \alpha u\sin\varphi$$

$$u\cos\varphi = \frac{dx}{dt}$$

$$u\sin\varphi = \frac{dy}{dt}$$

$$\frac{d^2x}{dt^2} = -\frac{\alpha}{m}\frac{dx}{dt}$$

[illegible]

$$\frac{d^2y}{dt^2} = -g - \frac{\alpha}{m}\frac{dy}{dt}$$

I Art.

$$\frac{dx}{dt} = p \qquad \frac{dy}{dt} = q \qquad \beta = \frac{\alpha}{m}$$

$$\frac{dp}{dt} = -\beta p \qquad\Big|\qquad \frac{dp}{p} = -\beta\,dt$$

$$\frac{dq}{dt} = -g - \beta q \qquad\Big|\qquad \frac{dq}{g+\beta q} = -dt$$

$\log p = -\beta t + C$

$p = e^{-\beta t + C}$ $\qquad e^{C} = A$

$= A e^{-\beta t}$

$\log(g + \beta q) = -\beta t + C'$

$g + \beta q = e^{-\beta t + C'}$

$= B e^{-\beta t}$

$t = 0 \mid p = a$

$A = a$

$p = a e^{-\beta t}$

$t = 0 \mid q = b$

$g + \beta b = B$

$q = \left(b + \frac{g}{\beta}\right) e^{-\beta t} - \frac{g}{\beta}$

$t = \infty$ $\lim p = 0$ [illegible] ~~Asymptote~~

$\lim q = -\frac{g}{\beta}$ [illegible] constant

[illegible]

$\sigma/ = ?$

$$0 = (b + g/\beta)\, e^{-\beta t} - g/\beta$$

$$b + g/\beta = g/\beta\; e^{\beta t}$$

$$e^{\beta t} = \frac{\beta b}{g} + 1$$

$$\beta t = \log\left[1 + \frac{\beta b}{g}\right]$$

$$\frac{\beta b}{g} + 1 = 1 + \beta t + \frac{\beta^2 t^2}{2}$$

$$t + \frac{\beta t^2}{2} = \frac{b}{g} \qquad \overline{\beta = 0}$$

$$t = \frac{b}{g} = \sigma / 10$$

$\sigma^2 \circ v^{o} \,{}^{2} < \rho\sigma /$ mm !

$$\frac{dx}{dt} = A e^{-\beta t}$$

$$x = -\frac{A}{\beta} e^{-\beta t} + C$$

$t = 0 \mid x = 0$

$$C = \frac{A}{\beta} \qquad x = \frac{A}{\beta}\left[1 - e^{-\beta t}\right]$$

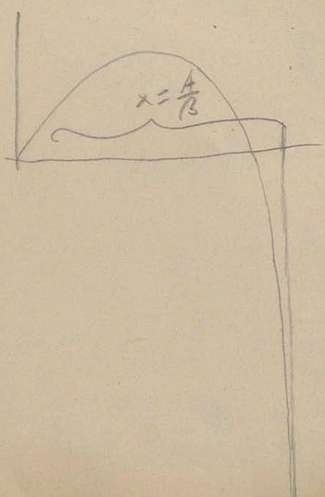
$$x = \frac{A}{\beta}\left[1 - \frac{1}{1+\frac{\beta b}{g}}\right]$$

$$= \frac{A}{\beta} \; \frac{\frac{\beta b}{g}}{1+\frac{\beta b}{g}}$$

$$x = \frac{\frac{A b}{g}}{1+\frac{\beta b}{g}} =$$

$\beta = 0 \quad x \ldots$

$k = \infty \quad x = \frac{A}{\beta}$



$$\frac{dy}{dt} = \left(b + \frac{g}{\beta}\right) e^{-\beta t} - \frac{g}{\beta}$$

$$y = -\frac{1}{\beta}\left(b + \frac{g}{\beta}\right) e^{-\beta t} - \frac{gt}{\beta} + D$$

$t = 0 \mid y = 0$

$$0 = -\frac{1}{\beta}\left(b + \frac{g}{\beta}\right) + D$$

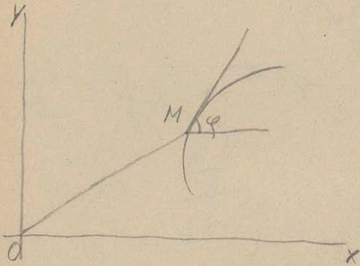
$$y = \frac{1}{\beta}\left[b + \frac{g}{\beta}\right]\left[1 - e^{-\beta t}\right] - \frac{gt}{\beta}$$

S: $y = 0$

$y = Kx - \log \ldots x$

quasi logarithmische Kurve

2/12 [illegible] algebr. [illegible]



$u^2 = p^2 + q^2$

$f_x = m \frac{dp}{dt}$

$\alpha f(u)$

$m \frac{dp}{dt} = - \alpha f(u) \cos\varphi$

$m \frac{dq}{dt} = - mg - \alpha f(u) \sin\varphi$

$p = u \cos\varphi$

$q = u \sin\varphi$

$$m \frac{dp}{dt} = - \alpha \frac{f(u)}{u} p$$

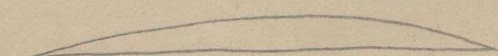
$$m \frac{dq}{dt} = - mg - \alpha \frac{f(u)}{u} q$$

$f(u) = u^2$

$$m \frac{dp}{dt} = - \alpha u p = - \alpha p \sqrt{p^2 + q^2}$$

[illegible] p [illegible] q [illegible] irrat.

[illegible] vert. [illegible], [illegible] Elevation



[illegible]

[illegible]

$$\frac{\alpha}{m} = \beta$$

$$\frac{dp}{dt} = -\beta p^2$$

$$\frac{dq}{dt} = -g - \beta q \sqrt{p^2+q^2}$$

$$\frac{dq}{dt} = -g - \beta p q$$

$$-\frac{dp}{p^2} = +\beta\, dt$$

$$\frac{1}{p} = \beta t + A \qquad \Big|\ t = 0 \qquad p = a,\ q = b$$

$$\frac{1}{a} = A$$

$$\frac{1}{p} = \beta t + \frac{1}{a}$$

$$p = \frac{a}{1 + a\beta t}$$

$$\frac{dx}{dt} = \frac{a}{1 + a\beta t}$$

$$x = \frac{1}{\beta}\ {}^{e}\log\,(1 + a\beta t) + A'$$

$$A' = 0$$

$$\frac{dq}{dt} = -g - \frac{\beta a q}{1 + a\beta t}$$

$$(1 + a\beta t)\frac{dq}{dt} + \beta a q = -g(1 + a\beta t)$$

[[illegible]]

$$\frac{d}{dt}\left[(1 + a\beta t)q\right] = -g(1 + a\beta t)$$

$$(1 + a\beta t)q = -g\left(t + \frac{a\beta t^2}{2}\right) + B$$

$$b = B$$

$$q = \frac{b}{1 + a\beta t} - g\,\frac{t + \frac{a\beta t^2}{2}}{1 + a\beta t}$$

$$\frac{dy}{dt} = \frac{b}{1 + a\beta t} - g\,\frac{\left[\frac{t}{2} + \frac{t}{2} + \frac{a\beta t^2}{2}\right]}{1 + a\beta t}$$

$$= \frac{b}{1 + a\beta t} - g\frac{t}{2} - \frac{g}{2}\,\frac{t}{1 + a\beta t}$$

$$\frac{dy}{dt} = \frac{b}{1+a\beta t} - g\frac{t}{2} - \frac{g}{2a\beta}\,\frac{a\beta t+1-1}{1+a\beta t}$$

$$= \frac{b}{1+a\beta t} - g\frac{t}{2} - \frac{g}{2a\beta} \cdots\cdots$$

$$= \left[b + \frac{g}{2a\beta}\right]\frac{1}{1+a\beta t} - g\frac{t}{2} - \frac{g}{2a\beta}$$

$$y = \frac{1}{a\beta}\left[b + \frac{g}{2a\beta}\right]\log(1+a\beta t) - g\frac{t^2}{4} - \frac{g}{2a\beta}t + B$$

$$B' = 0$$

[illegible] ?

$$q = 0$$

$$b = +g\left[t + \frac{a\beta t^2}{2}\right]$$

$$\frac{b}{g}\,\frac{2}{a\beta} = \frac{2}{a\beta}t + t^2$$

$$t = -\frac{1}{a\beta} \pm \sqrt{\frac{1}{a^2\beta^2} + \frac{2b}{a\beta g}}$$

[illegible]

$$1 + a\beta t = \sqrt{1 + \frac{2ab\beta}{g}}$$

$$x = \frac{1}{\beta} \log \left(1 + a\beta t\right).$$

$$= \frac{1}{2\beta} \log \left[1 + \frac{2ab\beta}{g}\right]$$

$$\boxed{ab = c^2 \sin\varphi \cos\varphi}$$

$$\beta = 0$$

$$x = \frac{0}{0} \text{ — [illegible]}$$

$$x = \frac{ab}{g}$$

[illegible] $y = 0$ [illegible] alg. [illegible]

$q = b$ [illegible]

horiz. [illegible] x [illegible] 0 [[illegible] $f(x) = u$ [illegible]]

[illegible]

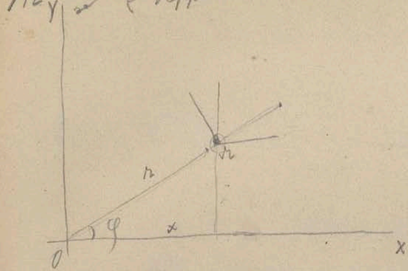
[illegible]

[illegible]

[illegible] $1 + a\beta t = 1000$ [illegible]

[illegible] $\log \ldots = 3$

3/12 [illegible] Keppler'schen [illegible] Gravitat.



[illegible]

$$m\frac{d^2x}{dt^2} = K\cos\varphi$$

$$= K\frac{x}{r} \qquad K = -\frac{m}{r^2}\cdot\alpha$$

$$= -\frac{\alpha m}{r^2}$$

$$m\frac{d^2x}{dt^2} = -\frac{\alpha m}{r^2}\frac{x}{r}$$

II		I
$\frac{dx}{dt}$	$\frac{d^2x}{dt^2} = -\frac{\alpha x}{r^3}$	$-y$
$\frac{dy}{dt}$	$\frac{d^2y}{dt^2} = -\frac{\alpha y}{r^3}$	x

[illegible]

$$x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2} = 0$$

$$\frac{d}{dt}\left(x\frac{dy}{dt} - y\frac{dx}{dt}\right) = 0$$

$$x\frac{dy}{dt} - y\frac{dx}{dt} = (2c)$$

[illegible]

I. 1 : [illegible]

[illegible]

[illegible] Centralb. [illegible]

$$\frac{dx}{dt}\frac{d^2x}{dt^2} + \frac{dy}{dt}\frac{d^2y}{dt^2} = -\frac{\alpha}{r^3}\left[x\frac{dx}{dt} + y\frac{dy}{dt}\right]$$

$$r^2 = x^2 + y^2$$

$$2r\frac{dr}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{d}{dt}\left[\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\left(\frac{dy}{dt}\right)^2\right] = \frac{\alpha}{r^2}\frac{dr}{dt}$$

$$= \frac{d}{dt}\left(\frac{\alpha}{r}\right)$$

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\left(\frac{dy}{dt}\right)^2 = \frac{\alpha}{r} + \beta$$

$$\frac{u^2}{2} = \frac{\alpha}{r} + \beta$$

[illegible]

$$m\frac{u^2}{2} = \frac{m\alpha}{r} + \cdots$$

$$p\frac{ds}{dt} = 2c$$ [illegible] p = Perpendi

$$u^2 = \left(\frac{ds}{dt}\right)^2 \qquad \left(\frac{ds}{dt}\right)^2 = \frac{4c^2}{p^2}$$

$$\frac{4c^2}{p^2} = \frac{2\alpha}{r} + 2\beta$$ [illegible]

$$u^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2$$

$$\frac{r^2 d\varphi}{dt} = 2c$$ [illegible]

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = \frac{2\alpha}{r} + 2\beta$$

$$\left(\frac{dr}{dt}\right)^2 + r^2\frac{4c^2}{r^4} = \dots$$

$$\left(\frac{dr}{dt}\right)^2 = 2\beta + \frac{2\alpha}{r} - \frac{4c^2}{r^2}$$

[illegible]

$$\frac{2\alpha}{r} = 2\frac{2c}{r}\, y$$

$y = \frac{1}{\rho}$ Coordinaten!

$$y = \frac{\alpha}{2c}$$

$$\left(\frac{dr}{dt}\right)^2 = 2\beta + \frac{\alpha^2}{4c^2} - \left(\frac{\alpha}{2c} - \frac{2c}{r}\right)^2$$

[illegible]

[illegible]

$t_0 = 0$ $\quad u_0 = $

$$u_0^2 = \frac{2\alpha}{r_0} + \beta$$

$$\frac{2\alpha}{r_0} + 2\beta \;!$$

$$\beta = u_0^2 - \frac{2\alpha}{r_0}$$

[illegible shorthand]

Kreisbahn

$$2\beta + \frac{\alpha^2}{4c^2} = 0$$

[illegible shorthand]

$$\left(\frac{dr}{dt}\right)^2 = -\left(\frac{\alpha}{2c} - \frac{2c}{r}\right)^2$$

[illegible shorthand]

$$\frac{\alpha}{2c} - \frac{2c}{r} = 0$$

[illegible shorthand]

$$r = \frac{4c^2}{\alpha}$$

$$u_0^2 - \frac{2\alpha}{r_0} + \frac{\alpha^2}{4c^2} = 0$$

$r = r_0$

$$\frac{\alpha^2}{4c^2} = \frac{\alpha}{r_0}$$

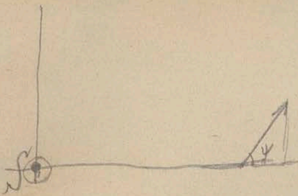
$$u_0^2 = \frac{\alpha}{r_0}$$

$$\frac{\alpha^2}{4c^2} = u_0^2 = \frac{\alpha}{r_0}$$

$$\frac{\alpha}{4c^2} = \frac{1}{r_0}$$

$$r_0 = \frac{4c^2}{\alpha}$$

[illegible shorthand]



$u_0^2 = \frac{\alpha}{r_0}$

$4c^2 = \alpha r_0$.

$2c = \frac{r\, u_0 \sin\psi}{[illegible]}$

$u_0^2 r_0^2 = \alpha r_0$.

$= 4c^2$

$2c = u_0 r_0$

$\sin\psi = 1$. [illegible]

[illegible]

$\frac{u_0^2}{r_0} = \left(\frac{\alpha}{r_0}\right)^2$ [illegible]

$$\frac{dr}{dt} = \sqrt{2\beta + \frac{\alpha^2}{4c^2} - \left(\frac{\alpha}{2c} - \frac{2c}{r}\right)^2}$$

$$\frac{d\varphi}{dt} = \frac{2c}{r^2}$$ r als f. v. φ φ_0

$$\frac{dr}{dt} = \frac{dr}{d\varphi} \cdot \frac{d\varphi}{dt}$$

$$= \frac{2c}{r^2} \frac{dr}{d\varphi}$$

$$\frac{2c}{r^2} \frac{dr}{d\varphi} = \pm \sqrt{- - - - -}$$

alle Bw. v. Arg φ fr



φ_0 ⊙ prst.

$$\frac{\frac{2c}{r^2} dr}{\sqrt{2\beta + \frac{\alpha^2}{4c^2} - \left(\frac{\alpha}{2c} - \frac{2c}{r}\right)^2}} = d\varphi$$

$$\left(\frac{\alpha}{2c} - \frac{2c}{r}\right) = z$$

$$\frac{2c}{r^2} dr = dz$$

$$\left(2\beta + \frac{\alpha^2}{4c^2}\right) = \rho^2$$

$$\frac{dz}{\sqrt{\rho^2 - z^2}} = d\varphi$$

$$\arcsin \frac{z}{\rho} = \varphi + A$$

$$\frac{z}{\rho} = \sin(\varphi + A)$$

$$\frac{\alpha}{2c} - \frac{2c}{r} = \rho \sin(\varphi + A)$$

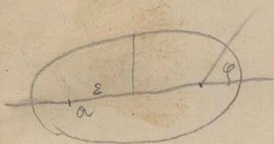
$$\frac{\alpha}{4c^2} - \frac{\gamma \sin(\varphi + A)}{2c} = \frac{1}{r}$$

$$r = \frac{1}{\frac{\alpha}{4c^2} - \frac{\gamma}{2c}\sin(\varphi + A)}$$ [illegible]

$$r = \frac{a(1-\varepsilon^2)}{1 + \varepsilon \cos\varphi}$$

$$r = \frac{\frac{4c^2}{\alpha}}{1 - \frac{2c\gamma}{\alpha}\sin(\varphi + A)}$$

[illegible]



$$\varepsilon = \frac{b}{a}$$

[illegible]

[illegible]

$$r = \frac{\frac{4c^2}{\alpha}}{1 - \frac{2c\gamma}{\alpha}\sin A}$$

[illegible] $\sin A = -1$

$$A = \frac{3\pi}{2}$$

$$\sin(\varphi + A) = \sin\left(\varphi + \frac{3\pi}{2}\right) =$$

$$= -\cos\varphi$$

$$= \frac{\frac{4c^2}{\alpha}}{1 + \frac{2c\gamma}{\alpha}\cos\varphi}$$

$$\gamma = \sqrt{2\beta + \frac{\alpha^2}{4c^2}} \qquad \frac{2c\gamma}{\alpha} = \sqrt{1 + \frac{8c^2\beta}{\alpha^2}}$$

Ellipse a $\frac{2c\mu}{\alpha} < 1$

Parabel $\frac{2c\mu}{\alpha} = 1$

Hyperbel $\frac{2c\mu}{\alpha} > 1$

[$\frac{2c\mu}{\alpha} = 0$]

$\frac{2c\mu}{\alpha} < 1$

$\frac{4c^2\mu^2}{\alpha^2} < 1$ $\frac{4c^2\mu^2}{\alpha^2} = 1$ $\frac{4c^2\mu^2}{\alpha^2} > 1$

$\frac{8\beta c^2}{\alpha^2} + 1 < 1$ $= 1$ > 1

$\frac{8\beta c^2}{\alpha^2} < 0$

$= 0$

> 0

Ell. Par. Hyperb.

$\beta -$ $\beta = 0$ $\beta +$

$u^2 = \frac{2\alpha}{r} + 2\beta$

$u_0^2 = \frac{2\alpha}{r_0} + 2\beta$ [$u_0^2 - \frac{2\alpha}{r_0} = 2\beta$

[illegible]

[illegible] r_0 [illegible] $\leqq \frac{2\alpha}{r_0}$

[illegible]

[illegible] u_0^2 [illegible]

[illegible] ∞ [illegible]

II K. S.

$$a(1-\varepsilon^2) = \frac{4c^2}{\alpha} \qquad a^2(1-\varepsilon^2) = \frac{4c^2a}{\alpha}$$

$$\varepsilon = \frac{2c\gamma}{\alpha} \qquad \varepsilon = \frac{e}{a}$$

$$\underbrace{a^2 - e^2}_{b^2} = \frac{4c^2a}{\alpha}$$

$c =$ [illegible]

$$cT = ab\pi \qquad c = \frac{\pi ab}{T} \qquad b^2 = \frac{4a}{\alpha}\,\frac{\pi^2a^2b^2}{T^2}$$

$$1 = \frac{4\pi}{\alpha}\,\frac{a^3}{T^2}$$

[illegible] K. S. [illegible]

[illegible]

[illegible] 2 [illegible] 8'' : [illegible] S [illegible]

[illegible] Newton'sche [illegible] 2^2 [illegible]

[illegible]

[illegible] Kometen [illegible] Ell. [illegible]

[illegible] Ell. [illegible]

[illegible] Astron. [illegible] Par. [illegible]

[illegible] const. [illegible] Ell. [illegible] Hyp.

[illegible] Parab. [illegible]

Curve [illegible] F [illegible]

[illegible]

$PM = u_0 \tau$



u_0 [illegible]

$u_0^2 = \frac{\alpha}{r}$ [illegible]

[illegible] ψ:

$$\Delta SPM = \frac{r_0 u_0 \tau \sin\psi}{2}$$

$$\frac{\Delta SPM}{\tau} = c = \frac{r_0 u_0 \sin\psi}{2}$$

$$2c = r_0 u_0 \sin\psi$$

$$\frac{4c^2}{\alpha} = \frac{r_0^2 u_0^2 \sin^2\psi}{\alpha}$$

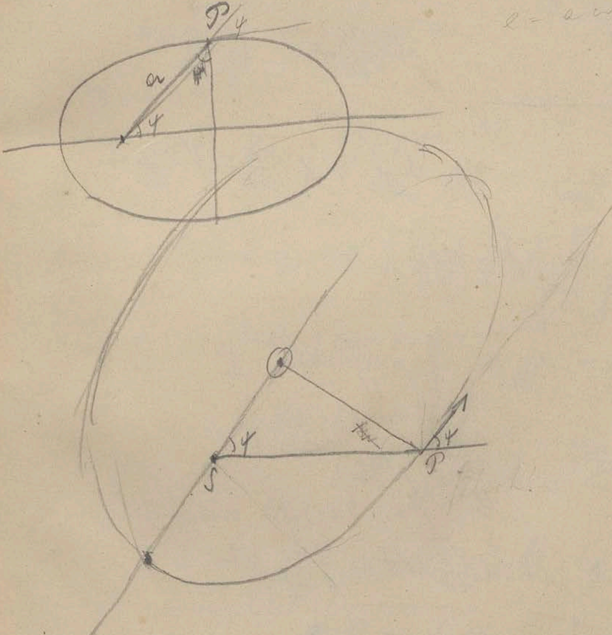
$$= r_0 \sin^2\psi = a\left(1-\varepsilon^2\right)$$

$$a\left(1-\varepsilon^2\right) = r_0\left(1-\cos^2\psi\right)$$

~~r_0~~ $a = r_0$

$\varepsilon = \cos\psi$

$e = \cos\psi$

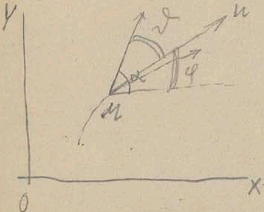


5/12 ve e: L.K. = ✓

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = X\,dx + Y\,dy$$

$$= p\,ds\cos\vartheta$$

$$m\frac{u^2}{2} = \frac{\alpha m}{r} + C$$



$$m\frac{d^2x}{dt^2} = X \quad \Big| \quad \frac{dx}{dt}$$

$$m\frac{d^2y}{dt^2} = Y \quad \Big| \quad \frac{dy}{dt}$$

$$m\frac{dx}{dt}\frac{d^2x}{dt^2} + m\frac{dy}{dt}\frac{d^2y}{dt^2} = X\frac{dx}{dt} + Y\frac{dy}{dt}$$

$$\frac{d}{dt}\left[\frac{m}{2}\left(\frac{dx}{dt}\right)^2 + \frac{m}{2}\left(\frac{dy}{dt}\right)^2\right] = X \; - - - -$$

$$u^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \qquad [u = ry \ldots]$$

$$\frac{d\left(m\frac{u^2}{2}\right)}{dt} = X\frac{dx}{dt} + Y\frac{dy}{dt};$$

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = X\frac{dx}{dt}\cdot dt + Y\frac{dy}{dt}\,dt$$

[illegible] = L.K. $\quad = X\,dx + Y\,dy$

Discussion:

[illegible] : $Y = 0$ $\quad d(l.k.) = X\,dx$

[illegible] l.k. [illegible] dt = [illegible]

[illegible] ([illegible] l.k. $\frac{mu^2}{2}$)

[illegible] Summe [illegible] X [illegible] Y

$$\left.\begin{array}{l} X = P\cos\alpha \\ Y = P\sin\alpha \end{array}\right\} \text{[illegible]}\ P$$

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = P\left(\cos\alpha\,\frac{dx}{dt} + \sin\alpha\,\frac{dy}{dt}\right)dt$$

$$\frac{ds}{dt} = u\,; \quad \frac{dx}{dt} = \frac{ds}{dt}\cos\varphi \;\Big|\; \frac{dy}{dt} = \frac{ds}{dt}\sin\varphi$$

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = P\frac{ds}{dt}\left[\cos\alpha\cos\varphi + \sin\alpha\sin\varphi\right]dt$$

$$= P\frac{ds}{dt}\,dt\,\cos\underbrace{(\alpha-\varphi)}_{\vartheta}$$

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = P\frac{ds}{dt}\,dt\cos\vartheta$$

$$= P\,ds\cos\vartheta = P\cos\vartheta\cdot ds$$

[illegible] ($\sphericalangle\,\vartheta$) [illegible] = [illegible] Compon.

[illegible] ($P\cos\vartheta$) [illegible] (ds)

$\vartheta = 0$; [illegible] ([illegible] $= P\,ds$

$\vartheta = \frac{\pi}{2}$ — — $= 0$)

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = X\,dx + Y\,dy$$

(X, Y ~ Newton. I II)

$$\frac{\alpha m}{r^2} = f = P$$

$$X = -\frac{\alpha m}{r^2}\left(\frac{x}{r}\right);\quad Y = \frac{\alpha m}{r^2}\,\frac{y}{r}$$

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = -\frac{\alpha m}{r^3}\left[x\,dx + y\,dy\right]$$

$r^2 = x^2 + y^2$

$$= -\frac{\alpha m}{r^2}\,dr \qquad r\,dr = x\,dx + y\,dy$$

$$= \alpha m\, d\left(\frac{1}{r}\right) = d\left(\frac{\alpha m}{r}\right)$$

$$\frac{mu^2}{2} = \frac{\alpha m}{r} + C$$

---

4. $P = f(r)$ $\quad X = f(r)\,\frac{x}{r}$ $\quad Y = f(r)\,\frac{y}{r}$

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = \frac{f(r)}{r}\left[x\,dx + y\,dy\right]$$

$$= f(r)\,dr$$

$$\frac{mu^2}{2} = \int f(r)\,dr \qquad d\,F(r) = f(r)\,dr$$

$$\frac{d}{dt}\left(\frac{mu^2}{2}\right)dt = d\,F(r)$$

$$\frac{mu^2}{2} = F(r) + C$$

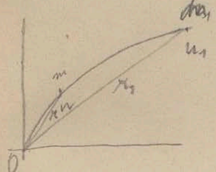
z. B. $P = \beta \frac{m}{r^3} = f(r)$; $F(r) = -\frac{1}{2} \beta \frac{m}{r^2}$

$$\frac{mu^2}{2} = -\beta \frac{m}{2r^2} + C$$

$$\frac{mu^2}{2} = F(r) + C$$

$$\frac{mu_1^2}{2} = F(r_1) + C$$

$$\frac{mu_1^2}{2} - \frac{mu^2}{2} = F(r_1) - F(r)$$



[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$\Phi = f(r)$ $\quad X = f(r) \frac{x}{r}$

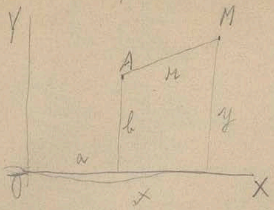
$$X = \frac{dF(r)}{dr} \cdot \frac{x}{r}; \quad Y = \frac{dF(r)}{dr} \frac{y}{r}$$

$r^2 = x^2 + y^2$; $\frac{r\,dr}{dx} = x$ $\quad \frac{dr}{dx} = \frac{x}{r}$ (partielles Diff. [illegible])

$$\frac{dr}{dy} = \frac{y}{r} \qquad X = \frac{dF(r)}{dr} \cdot \frac{dr}{dx}$$

$X = \frac{d F(r)}{dx}$; $y = \frac{d F(r)}{dy}$

$\frac{\partial F(r)}{\partial x}$ $\frac{\partial F(r)}{\partial y}$



$f(r)$ [illegible]

$$m \frac{d^2x}{dt^2} = X$$

$$X = f(r) \cos\alpha$$

$$r^2 = (x-a)^2 + (y-b)^2$$

$$\cos\alpha = \frac{x-a}{r}$$

$$X = f(r) \frac{x-a}{r}$$

[illegible] $F(r)$ [illegible]

$$f(r) = \frac{d(F(r))}{dr}$$

diff

$$r \frac{dr}{dx} = x - a$$

$$\frac{dr}{dx} = \frac{(x-a)}{r} = \cos\alpha$$

$$X = \frac{d(F(r))}{dr} \frac{dr}{dx}$$

$$= \frac{d F(r)}{dx}$$

[illegible] 120 [illegible]

$\frac{dx}{dt}\Big|$ $m \frac{d^2x}{dt^2} = \frac{d F(r)}{dx}$ $\qquad m \frac{d^2y}{dt^2} = \frac{d F(r)}{dy}$ $\Big| \frac{dy}{dt}$

$$\frac{d}{dt}\left[\frac{m}{2}\left(\frac{dx}{dt}\right)^2 + \frac{m}{2}\left(\frac{dy}{dt}\right)^2\right] = \frac{dF}{dx}\frac{dx}{dt} + \frac{dF}{dy}\frac{dy}{dt}$$

$F = F(r)$ ges

$$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt = \frac{dF}{dx}\frac{dx}{dt}dt + \frac{dF}{dy}\frac{dy}{dt}dt$$

$$= \frac{dF}{dx}dx + \frac{dF}{dy}dy$$

$\frac{d}{dt}\left(m\frac{u^2}{2}\right)dt =$ totale [illegible] F = totale Diff

$$= dF$$

$$m\frac{u^2}{2} = F + C$$

$r = r_0 \quad u = u_0$

$$m\frac{u_0^2}{2} = F(r_0) + C$$

$$m\frac{u^2}{2} = F(r) + C$$

$$m\frac{u^2}{2} - m\frac{u_0^2}{2} = F(r) - F(r_0)$$

[illegible] | [illegible] Kraftfunction

[illegible] U [illegible]

$$X = \frac{dU}{dx}$$

$$Y = \frac{dU}{dy}$$

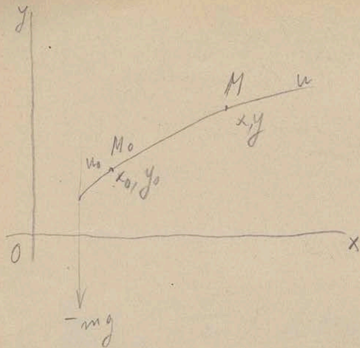
$$\frac{dx}{dt}\Big|\ m\frac{d^2x}{dt^2} = \frac{dU}{dx} \qquad m\frac{d^2y}{dt^2} = \frac{dU}{dy}\ \Big|\ \frac{dy}{dt}$$

$$\frac{d}{dt}\left(m\frac{v^2}{2}\right)dt = \underbrace{\frac{dU}{dx}dx + \frac{dU}{dy}dy}_{dU}$$

$$m\frac{v^2}{2} = U + C$$

$$m\frac{v_0^2}{2} = U_0 + C$$

$$m\frac{v^2}{2} - m\frac{v_0^2}{2} = U - U_0$$



[illegible]

$$-mg$$

$$U = -mgy$$

$$\frac{dU}{dx} = 0$$

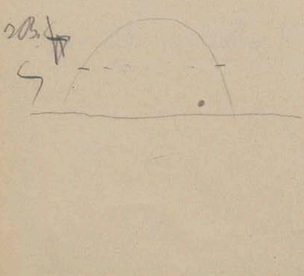
$$\frac{dU}{dy} = -mg$$

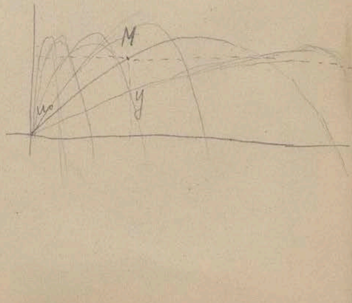
[illegible]

$$m\frac{u^2}{2} - m\frac{u_0^2}{2} = -mgy + mgy_0$$

$$= -mg[y - y_0]$$

[illegible]







$$X = -kx$$

$$U = -\frac{kx^2}{2}$$

$$\frac{dU}{dx} = -kx$$

$$\frac{dU}{dy} = 0$$

[illegible]

[illegible] U [illegible] physik. [illegible]

$$\frac{d}{dt}\left(\frac{mu^2}{2}\right)dt = X\,dx + Y\,dy = \frac{dU}{dx}dx + \frac{dU}{dy}dy$$

[illegible] $= dU$

[illegible]

totale [illegible] Kraft [illegible] $=$ [illegible]

[illegible] Kraft [illegible] neg. [illegible]

$$U = -W$$

$$X = \frac{dU}{dx} = -\frac{dW}{dx}$$

$$m\frac{u^2}{2} - m\frac{u_0^2}{2} = -W + W_0$$

[illegible]

$$m\frac{u^2}{2} + W = m\frac{u_0^2}{2} + W_0$$

constante ?

Energie

$$m\frac{u^2}{2} + W = \text{const}$$

[illegible] ^{2} — const. ?

[[illegible]]

[illegible] W [illegible] Energie = potentielle Energie

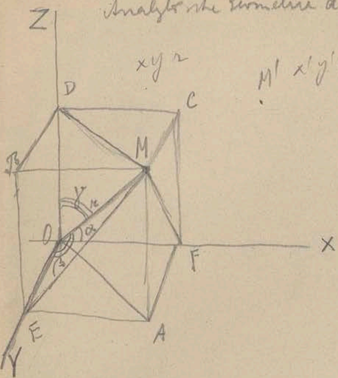
[illegible] = Energie

[illegible] Energie [illegible]

[illegible] Energie

10/12

Analytische Geometrie d. Raumes



xyz $\quad M'\ x'y'z'$

MB = Abscisse = X Coord. $= OF$

MC = Y Ord. $= OE$

MA = Z Coord. $= OD$

$$r^2 = OA^2 + MA^2 = OA^2 + Z^2$$

$$= OF^2 + FA^2 + Z^2$$

$$= x^2 + y^2 + z^2$$

$x = r\cos\alpha$

$y = r\cos\beta$

$z = r\cos\gamma$

$$x^2+y^2+z^2 = r^2\left[\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right]$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

[v. M' ~ ρ, e Coordination]

$$R^2 = \rho^2 = (x'-x)^2 + (y'-y)^2 + (z'-z)^2$$

$x'-x = R\cos a$
$y'-y = R\cos b$
$z'-z = R\cos c$

$x' = r'\cos\alpha'$

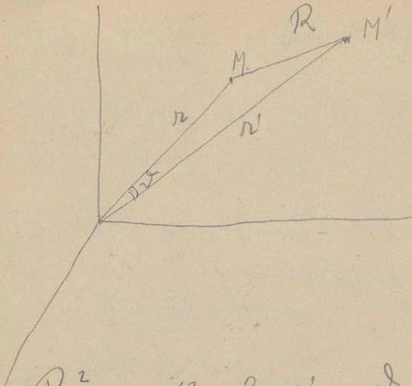
$y' = r'\cos\beta'$

$z' = r'\cos\gamma'$

$$x'^2 + y'^2 + z'^2 = R'^2$$

$$R^2 = x'^2 + y'^2 + z'^2 + x^2 + y^2 + z^2 - 2(x'x + y'y + z'z)$$

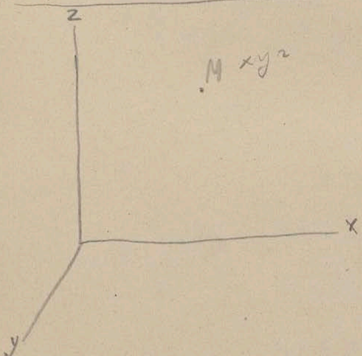
$$R^2 = R'^2 + R^2 - 2Rr'(\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma')$$



$R^2 = r^2 + r'^2 - 2rr' \cos\vartheta$

$R^2 = r^2 + r'^2 - 2rr'(\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma')$

$\cos\vartheta = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'$



$m\frac{d^2x}{dt^2} = X$ | $\frac{dx}{dt}$

$m\frac{d^2y}{dt^2} = Y$ | $\frac{dy}{dt}$

$m\frac{d^2z}{dt^2} = Z$ | $\frac{dz}{dt}$

$P \mid \mathfrak{A}\ \mathfrak{B}\ \mathfrak{C}$

$X = P\cos\mathfrak{A}$

$Y = P\cos\mathfrak{B}$

$Z = P\cos\mathfrak{C}$

$$m\left(\frac{dx}{dt}\frac{d^2x}{dt^2}+\frac{dy}{dt}\frac{d^2y}{dt^2}+\frac{dz}{dt}\frac{d^2z}{dt^2}\right)=X\frac{dx}{dt}+\text{------}$$

$$\frac{d}{dt}\left[\frac{m}{2}\left(\frac{dx}{dt}\right)^2+\text{------}\right]dt=\left(X\frac{dx}{dt}+Y\frac{dy}{dt}+Z\frac{dz}{dt}\right)$$

$$\left(\frac{dx}{dt}\right)^2+(\ \ )^2+(\ \ )^2=c^2$$

$$\frac{d}{dt}\left(\frac{m}{2}c^2\right)dt \qquad = \underbrace{X\,dx+Y\,dy+Z\,dz}$$

[illegible]

[illegible]

[illegible]

$X=P\cos A \qquad Y=P\cos B \qquad Z=P\cos C$

$dx = ds\cos f$

$dy = ds\cos g$

$dz = ds\cos h$

$$X\,dx+Y\,dy+Z\,dz=P\,ds\left[\cos A\cos f+\cos B\cos g+\cos C\cos h\right]$$

$$=P\,ds\cos\varphi$$

φ [illegible]

[illegible]

[illegible] U [illegible]

$$X = \frac{dU}{dx} \qquad Y = \frac{dU}{dy} \qquad Z = \frac{dU}{dz}$$

$$\frac{d}{dt}\left(m\frac{c^2}{2}\right) = dU$$ [illegible] Diff. dU [illegible]

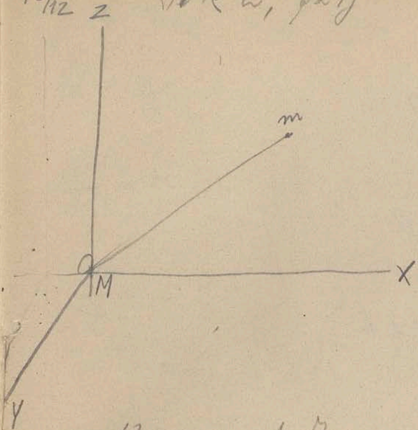
[illegible] Coord. [illegible]

[illegible]

$$m\frac{c^2}{2} = U + K$$

12/12 [illegible]

$$\frac{kMm}{r^2}$$



$$m\frac{d^2x}{dt^2} = -\frac{kMm}{r^2}\frac{x}{r}$$

m — — — — —

— — — — — —

$kM = \alpha$

$$\frac{d^2x}{dt^2} = -\frac{\alpha x}{r^3} \quad \Big| \; y$$

$$\frac{d^2y}{dt^2} = -\frac{\alpha y}{r^3} \quad \Big| \; x \quad \Big| \; z$$

$$\frac{d^2z}{dt^2} = -\frac{\alpha z}{r^3} \quad \Big| \quad \Big| \; y$$

$$x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2} = 0$$

$$\frac{d}{dt}\left(x\frac{dy}{dt} - y\frac{dx}{dt}\right) = 0 \qquad x\frac{dy}{dt} - y\frac{dx}{dt} = 2a$$

[illegible] Projectionsp. [illegible]

$$y\frac{d^2z}{dt^2} - z\frac{d^2y}{dt^2} = 0$$

$$\frac{d}{dt}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right) = 0$$

$$y\frac{dz}{dt} - z\frac{dy}{dt} = 2b$$

$$z\frac{d^2x}{dt^2} - x\frac{d^2z}{dt^2} = 0$$

$$z\frac{dx}{dt} - x\frac{dz}{dt} = 2c$$ [illegible] Projecti[illegible]

[illegible] xy [illegible] proj. [illegible] II K.S.

[illegible] yz [illegible] xz [illegible]

f $\quad$ $A\ B\ C$ } Proj. [illegible] f

$A = f\cos l$	$A^2+B^2+C^2 = f^2\underbrace{[\cos^2 l + \cos^2 m + \cos^2 n]}_{1}$
$B = f\cos m$	$= f^2$
$C = f\cos n$	

[illegible] f [illegible] const. [illegible] l, m, n const. [illegible]

[illegible] Curve.

[illegible]

$$U = \frac{\alpha}{r}$$

$$\frac{dU}{dx} = -\frac{\alpha}{r^2}\frac{dr}{dx} = -\frac{\alpha}{r^2}\frac{x}{r}$$

$$\frac{c^2}{2} = U + h$$

$$c^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2$$

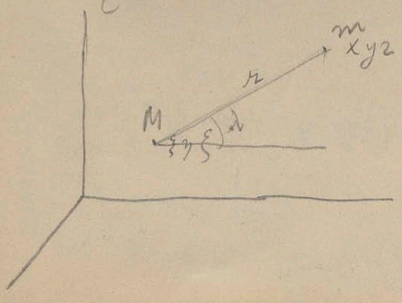
$$c_0 \quad | \; U_0 \; |$$

$$\frac{c^2}{2} - \frac{c_0^2}{2} = U - U_0$$

[illegible]

[illegible] Central [illegible]

[illegible]



$$m\frac{d^2x}{dt^2} = X$$

$$= -\frac{kMm}{r^2}\,\frac{x-\xi}{r}$$

$$\cos\lambda = \frac{x-\xi}{r}$$

[illegible]

$$m\frac{d^2x}{dt^2} = -\frac{kMm}{r^3}(x-\xi)$$

$$M\frac{d^2\xi}{dt^2} = \Xi$$

$$\Xi = \frac{kMm}{r^2}\,\frac{x-\xi}{r}$$

$$M\frac{d^2\xi}{dt^2} = \frac{kMm}{r^3}(x-\xi)$$

[illegible] Newton [illegible]

$$m\frac{d^2x}{dt^2} = -\frac{kMm}{r^3}(x-\xi) \qquad M\frac{d^2\xi}{dt^2} = \frac{kMm}{r^3}(x-\xi)$$

$$m\frac{d^2y}{dt^2} = -\frac{kMm}{r^3}(y-\eta) \qquad M\frac{d^2\eta}{dt^2} = \frac{kMm}{r^3}(y-\eta)$$

$$r^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$$

$$\frac{d^2x}{dt^2} = -\frac{kM}{r^3}(x-\xi) \qquad \frac{d^2\xi}{dt^2} = \frac{km}{r^3}(x-\xi)$$

$$\frac{d^2(x-\xi)}{dt^2} = -\frac{k}{r^3}(x-\xi)(M+m)$$

$$\frac{d^2(y-\eta)}{dt^2} = -\frac{k(M+m)}{r^3}(y-\eta)$$

[illegible]

$(M+m)$ [illegible] M

$$\left[\frac{d^2x}{dt^2} = -\frac{kM}{r^3}x\right]$$ [illegible] Coord. diff. [illegible]

[illegible]

[illegible] Ell. [illegible] Central [illegible]

[illegible]

[illegible] III K. 3.

[illegible]

[illegible]

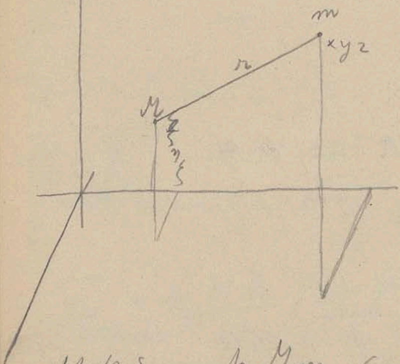
[illegible]

[illegible] 4 [illegible] constatierbar [illegible]

[illegible] $\frac{1}{500}$ [illegible]

16/12

[illegible]



$$M\frac{d^2\xi}{dt^2} = \frac{kMm}{r^3}(x-\xi) \qquad m\frac{d^2x}{dt^2} = -\frac{kMm}{r^3}(x-\xi)$$

$$M\frac{d^2\eta}{dt^2} = \qquad m\frac{d^2y}{dt^2} =$$

$$M\frac{d^2\zeta}{dt^2} = \qquad m\frac{d^2z}{dt^2} =$$

~~[illegible]~~

$$M\frac{d^2\xi}{dt^2} + m\frac{d^2x}{dt^2} = 0$$

$$M\frac{d\xi}{dt} + m\frac{dx}{dt} = A$$

$$\frac{d}{dt}\left[M\xi + mx\right] = A$$

$$M\xi + mx = At + A'$$

$$M\eta + my = Bt + B'$$

$$M\zeta + mz = Ct + C'$$

$$M\xi + mx = (M+m)\,a$$

$$a = \frac{M\xi + mx}{M+m}$$

$$b = \frac{M\eta + my}{M+m}$$

$$c = \frac{M\zeta + mz}{M+m}$$

[illegible] 3 [illegible] Coord. [illegible] = [illegible] [[illegible]]

$$M\frac{d^2\xi}{dt^2} + m\frac{d^2x}{dt^2} = 0$$

$$\frac{d^2}{dt^2}(M\xi + mx) = 0$$

$$\frac{d^2[(M+m)a]}{dt^2} = 0$$

$$(M+m)\frac{d^2a}{dt^2} = 0$$

[illegible] $\frac{d^2a}{dt^2} = 0$

[illegible] const. [illegible] a-linear f.

~~[illegible]~~

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$M\left(\zeta \frac{d^2\xi}{dt^2} - \xi \frac{d^2\zeta}{dt^2}\right) = k\frac{Mm}{r^3}\left(x\zeta - \xi\zeta - z\xi + \xi\zeta\right)$$

$$m\left(z \frac{d^2x}{dt^2} - x \frac{d^2z}{dt^2}\right) = -k\frac{Mm}{r^3}\left(x\zeta - \xi\zeta - zx + \zeta x\right)$$

$$M\left(\xi\frac{d^2\zeta}{dt^2}-\zeta\frac{d^2\xi}{dt^2}\right)+m\left(z\frac{d^2x}{dt^2}-x\frac{d^2z}{dt^2}\right)=0$$

$$=M\frac{d}{dt}\left(\xi\frac{d\zeta}{dt}-\zeta\frac{d\xi}{dt}\right)+m\frac{d}{dt}\left(z\frac{dx}{dt}-x\frac{dz}{dt}\right)=0$$

$$M\left(\xi\frac{d\zeta}{dt}-\zeta\frac{d\xi}{dt}\right)+m\left(z\frac{dx}{dt}-x\frac{dz}{dt}\right)=F$$

[illegible] RV [illegible]

proj. [illegible]

RVΩ [illegible] Coord. ‖ [illegible] X Ω [illegible] x y Ebene [illegible]

[illegible]

[illegible] RVΩ [illegible]

20 mult. – Coord. [illegible]

[illegible] [[illegible]]

$$r^2=(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2$$

$$X=-k\frac{Mm}{r^2}\left(\frac{x-\xi}{r}\right)$$

$$\frac{r\,dr}{dx}=x-\xi \qquad X=-k\frac{Mm}{r^2}\frac{dr}{dx}$$

[illegible] $\qquad = k\,Mm\,\frac{d}{dx}\left(\frac{1}{r}\right)$

$$\frac{d}{dx}\left(k\frac{Mm}{r}\right)$$

$$U = k\frac{Mm}{r}$$

[illegible] ⊙ :

$$\frac{dU}{d\xi} = -k\frac{Mm}{r^2}\frac{dr}{d\xi} = k\frac{Mm}{r^2}\frac{(x-\xi)}{r}$$

$$r^2 = \text{------}$$

$$r\frac{dr}{d\xi} = -(x-\xi)$$

[illegible] Coord. [illegible]

$$\frac{d\xi}{dt} \quad M\frac{d^2\xi}{dt^2} = \frac{dU}{d\xi}$$

$$m\frac{d^2x}{dt^2} = \frac{dU}{dx}$$

$$\frac{dx}{dt}$$

$$M\left[\frac{d\xi}{dt}\frac{d^2\xi}{dt^2}+ \text{———}\right]+$$

$$m\left[\frac{dx}{dt}\frac{d^2x}{dt^2}+\text{———}\right]=\frac{dU}{d\xi}\frac{d\xi}{dt}+\frac{dU}{dy}\frac{dy}{dt}+$$

$$+\frac{dU}{d\xi}\frac{d\xi}{dt}+\frac{dU}{dx}\frac{dx}{dt}+d\text{———}$$

$$\frac{d}{dt}\left\{\frac{M}{2}\left[\left(\frac{d\xi}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{d\xi}{dt}\right)^2\right]+\frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\text{———}\right]\right\}dt$$

$$=\frac{dU}{d\xi}d\xi+\frac{dU}{dy}dy+\ldots+\frac{dU}{dx}dx+\text{———}$$

[illegible] 6 Variabeln

[illegible]

$$= dU$$

[illegible]

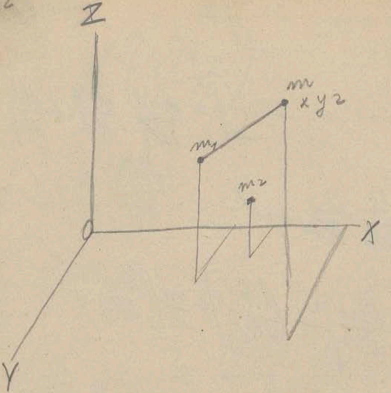
[illegible]

[illegible]

[illegible]

[illegible]

17/12



$$X_1 = -\frac{k\,m\,m_1}{r_1^2} \text{ [illegible] } \frac{x - x_1}{r_1}$$

$$X_2 = -\frac{k\,m\,m_2}{r_2^2}\,\frac{x - x_2}{r_2}$$

$$X = X_1 + X_2$$

$$U_1 = \frac{k\,m\,m_1}{r_1}$$

$$\frac{dU_1}{dx} = -\frac{k\,m\,m_1}{r_1^2}\,\frac{dr_1}{dx}$$

$$r_1^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2$$

r_1

$$r_1 \frac{dr_1}{dx} = x - x_1$$

$$U_2 = \frac{k\,m\,m_2}{r_2}$$

$$X_1 = \frac{dU_1}{dx}$$

$$X_2 = \frac{dU_2}{dx}$$

$$X_1 + X_2 = X = \frac{dU_1}{dx} + \frac{dU_2}{dx} =$$

$$= \frac{d}{dx}\left(U_1 + U_2\right)$$

[illegible]

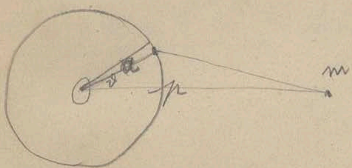
$u_1 + u_2 =$ [illegible]

[illegible]

[illegible]

[illegible]

Potential einer Kugelschale in Bezug auf einen Punkt



[illegible]

[illegible]

$\mu = \omega \sigma$ [illegible]

$$\frac{k m \mu}{r} = \frac{k m \omega \sigma}{r}$$

[illegible]

$a\, d\vartheta =$ [illegible]

$d\varphi$ [illegible]

$a \sin\vartheta\, d\varphi$

$f = a^2 \sin\vartheta\, d\vartheta\, d\varphi = \omega$

$$\frac{k\rho m \sigma a^2 \sin\vartheta \, d\vartheta \, d\varphi}{r}$$

r ~ [illegible] integr

$$2\pi \frac{k\rho m \sigma a^2 \sin\vartheta \, d\vartheta}{r}$$

[illegible]

$r^2 = a^2 + w$

$r^2 = a^2 + p^2 - 2ap\cos\vartheta$

$$\frac{2\pi k m \sigma a}{p} \frac{ap \sin\vartheta \, d\vartheta}{\sqrt{a^2+p^2-2ap\cos\vartheta}}$$

$$\frac{d}{d\vartheta}\sqrt{a^2+p^2-2ap\cos\vartheta}$$

$$\frac{1}{2}\frac{2ap\sin\vartheta \, \delta\vartheta}{\sqrt{\ \ }}$$

$$\int \frac{2\pi k m \sigma a}{p} d\sqrt{a^2+p^2-2ap\cos\vartheta}$$

$$\frac{2\pi k m \sigma a}{p}\sqrt{a^2+p^2-2ap\cos\vartheta}$$

$$\Big|_{\vartheta=0}^{\vartheta=\pi}$$

$$\frac{2\pi k m \sigma a}{p}\Big[(a+p)-(p-a)\Big]$$

$(a-p)$ [illegible] $(p-a)$: [illegible]

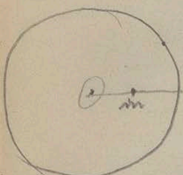
[illegible] $(p-a)$

$$= \frac{4\pi k m \sigma a^2}{p} = km\,\frac{4\pi a^2\sigma}{p}$$

$4\pi a^2\sigma =$ [illegible]

[illegible]

concentriert [illegible]



$$\Big|_{\vartheta=0}^{\vartheta=\pi}\ \frac{2\pi k m \sigma a}{r}\sqrt{a^2+p^2-2\ldots}$$

$$\frac{2\pi k m \sigma a}{r}\Big[(a+p)-(a-p)\Big]$$

$$= 4\pi k m \sigma a$$

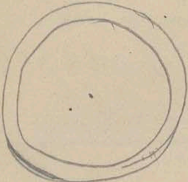
$$= \frac{4\pi a^2\sigma}{a}\,km$$

[illegible]

[illegible] $=0$; [illegible]

[illegible]

[illegible]



[illegible]

[illegible] Newton [illegible]

[illegible]

[illegible]

[illegible] $s = 5.6$

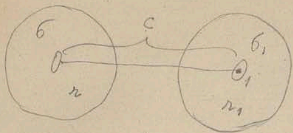
$M = \frac{4\pi R^3}{3} s$ [illegible]

$k \frac{M}{R^2} m = mg$

$k \frac{4\pi R s}{3} = g \qquad k = \frac{3g}{4\pi R s} = \frac{3g}{8 s \frac{\pi R}{2}}$

$= \frac{3g}{8 s\, 10^7} = \frac{3 \cdot 9.806}{8 \cdot 5.6 \cdot 10^7} = \frac{2}{3 \cdot 10^7}$ [illegible] 10^7 [illegible]

18/12



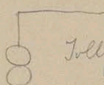
$$\frac{k\, m\, m_1}{c^2}$$

$$m = \frac{4\pi r^3}{3}\sigma$$

$$m_1 = \frac{4\pi r_1^3}{3}\sigma_1$$

$s = s_1$
$r = r_1$
$c = 2r$

[illegible] $\quad k\,\frac{4\pi^2 r^4 \sigma^2}{9} = 1$



Jolly's Versuch

$r = 1;\ s = 1$

$1 = k\,\frac{4\pi^2}{9}$

$\frac{v^2}{\rho}$

$\frac{2\pi\rho}{t} = v$ [illegible] $= \frac{4\pi^2\rho^2}{t^2\cdot\rho}$ [illegible]

$f = \frac{kM}{\rho^2}$ $\quad M = \frac{4\pi^2\rho^3}{t^2}$ $\left[\frac{1}{k}\right]$

$\frac{d^2x}{dt^2} = -\frac{\alpha x}{r^3}$; $\alpha = k.M$

$\frac{4\pi^2 a^3}{t^2} = \alpha$ [III Kepl. G.] $= k.M$ [illegible]

K [illegible]

[illegible]

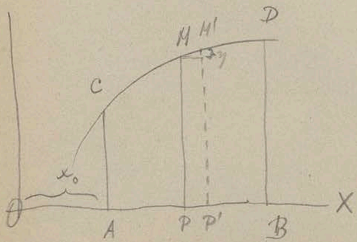
[illegible]

[illegible]

$$g = \frac{K.M}{R^2} \leftarrow \text{[illegible]}$$

$$\frac{4\pi^2 a^3}{T^2} = KM \qquad g R^2 = K.\mu$$

$$\frac{M}{\mu} = \frac{4\pi^2 a^3}{T^2 g R^2} \quad \text{[illegible]}$$



$y = f(x)$

$\overline{CABD}$ $\int$ $\Big\}$ $F(x)$

$\overline{APMC} = F(x)$

$$PP' = \xi \; ; \; F(x+\xi) - F(x) = y\xi + \frac{\eta \xi}{2}$$

$$\frac{F(x+\xi) - F(x)}{\xi} = y + \frac{\eta}{2}$$

$$\frac{d\,F(x)}{dx} = y \; [\lim] = f(x)$$

$$\underline{d\,F(x)} = f(x)\,dx \; ; \; \underline{F(x)} = \int f(x)\,dx$$

$$= \int y.\,dx + C$$

$F(x_0) = 0$; $\quad 0 = \int y\,dx\Big|_{x=x_0} + C$

$F(x) = \int y\,dx\Big|_{x=x} - \int y\,dx\Big|_{x=x_0}$

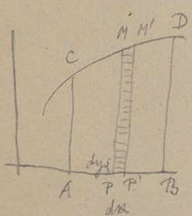
$F(x_1) = \int y\,dx\Big|_{x=x_1} - \int y\,dx\Big|_{x=x_0}$ [illegible]

$= \int_{x_0}^{x_1} y\,dx$ [illegible] [illegible]

synthetisch [illegible]

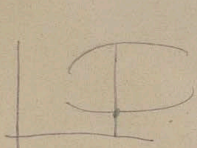
$y\,dx$ [illegible] $F(x_1) = \sum_{x_0}^{x_1} y\,dx$

[illegible] Summation



$\int_{x_0}^{x_1} dx \int_0^y dy$ = Fläche = $F(x_1)$?

$= \int_{x_0}^{x_1} dx\, y$



$\int_{x_0}^{x_1} dx \int_{y_0}^{y} dy = \int_{x_0}^{x_1} dx\,(y - y_0)$

(doppelte Int.) $F(x_1) = \int_{x_0}^{x_1}\int_{y_0}^{y_1} dx\,dy$

34463

20

III.

Dr. Josef Stefan

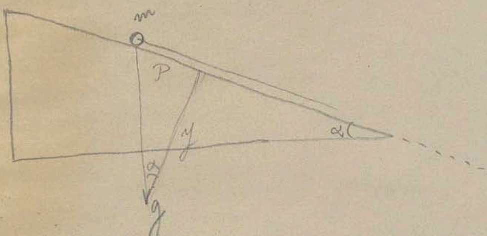
Mechanik

I. S. 90/91 M Smoluchowski

7/1 [illegible]

[illegible]

z.B. [illegible]



[illegible] Coord. [illegible]

$$m\frac{d^2s}{dt^2} = P = mg\sin\alpha$$

$$m\frac{d^2y}{dt^2} = mg\cos\alpha + N$$ [illegible] N [illegible] = 0 [illegible]

$$mg\cos\alpha + N = 0$$

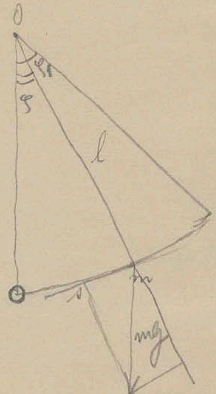
N [illegible]

[illegible] $mg\cos\alpha$ [illegible]

[illegible]

[illegible]

[illegible]



[illegible]

$$m\frac{d^2s}{dt^2} = -mg\sin\varphi$$

[illegible]

$$\frac{m v^2}{l} = mg\cos\varphi + S$$

[illegible]

$$\frac{d^2s}{dt^2} = -g\sin\varphi \qquad s = l\varphi$$

$$l\frac{d^2\varphi}{dt^2} = -g\sin\varphi$$

$$v = \frac{ds}{dt} = l\frac{d\varphi}{dt}$$

$$l\frac{d\varphi}{dt}\frac{d^2\varphi}{dt^2} = -g\sin\varphi\frac{d\varphi}{dt}$$

$$l\frac{d}{dt}\left[\frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2\right] = g\frac{d}{dt}(\cos\varphi)$$

$$\frac{l}{2}\left(\frac{d\varphi}{dt}\right)^2 = g\cos\varphi + C$$

[illegible]

$\varphi = 0$ $\quad v_0$

$$\frac{l}{2}\frac{v_0^2}{l^2} = g + C$$

$$\frac{1}{2l}\left(v_0^2 - v^2\right) = g\left(1 - \cos\varphi\right)$$

$$v_0^2 - v^2 = 2gl\,[1 - \cos\varphi]$$

$\cos\varphi < 1$ [illegible]

[illegible] $v = 0$ [illegible]

$$v_0^2 = 2gl\,[1 - \cos\varphi_1]$$

[illegible] v_0 [illegible] φ_1 [illegible]

[illegible]

[illegible]

$$\cos\varphi_1 = 1 - \frac{v_0^2}{2gl}$$

[illegible] v_0^2 [illegible] $\cos\varphi = 0$ [illegible]

[illegible] $\cos\varphi$ – [illegible]

$\varphi \sim 180°$ [illegible] $v_0^2 - 2gl = 2$

[illegible]

[illegible]

φ [illegible] $\cos\varphi > 1$ [illegible]

[illegible]

[illegible]

$$\frac{l}{2}\left(\frac{d\varphi}{dt}\right)^2 = g\cos\varphi + C$$

$$\varphi_1 \mid v_1 = 0$$

$$\frac{d\varphi}{dt} = 0$$

$$0 = g\cos\varphi_1 + C$$

$$\frac{l}{2}\left(\frac{d\varphi}{dt}\right)^2 = g[\cos\varphi - \cos\varphi_1]$$

$$\left(\frac{d\varphi}{dt}\right)^2 = \sqrt{\frac{2g}{l}[\cos\varphi - \cos\varphi_1]}$$

[illegible]

$$\frac{d\varphi}{dt} = \sqrt{\frac{2g}{l}}\sqrt{\cos\varphi - \cos\varphi_1}$$

$$\frac{d\varphi}{\sqrt{\cos\varphi - \cos\varphi_1}} = dt\sqrt{\frac{2g}{l}}$$

gibt elliptische Function

[illegible]

$$\cos\varphi = 1 - \frac{\varphi^2}{2} + \frac{\varphi^4}{24} - \cdots$$

$$\cos\varphi_1 = 1 - \frac{\varphi_1^2}{2} + \frac{\varphi_1^4}{2.3.4} - \cdots$$

[illegible]

$$\cos\varphi - \cos\varphi_1 = \frac{\varphi_1^2}{2} - \frac{\varphi^2}{2}$$

$$\frac{d\varphi}{\sqrt{\varphi_1^2 - \varphi^2}} = dt\sqrt{\frac{g}{l}}$$

$$\arcsin\left(\frac{\varphi}{\varphi_1}\right) = t\sqrt{\frac{g}{l}} + A$$

$$\varphi = 0 \quad |t=0|$$

$$\arcsin 0 = A \qquad A = 0$$

$$\frac{\varphi}{\varphi_1} = \sin\left(t\sqrt{\frac{g}{l}}\right)$$

$$\varphi = \varphi_1 \sin\left(t\sqrt{\frac{g}{l}}\right)$$

i) period.

φ_1 = Amplit. = ...

$t\sqrt{\frac{g}{l}} = \frac{\pi}{2}$

wenn $\varphi = \varphi_1$

$t_1 = \frac{\pi}{2}\sqrt{\frac{l}{g}}$

t_2 [...] $t_2\sqrt{\frac{g}{l}} = \pi$

$t_2 = \pi\sqrt{\frac{l}{g}}$

—

$t_3\sqrt{\frac{g}{l}} = 3\frac{\pi}{2}$

$\varphi = -\varphi_1 \mid t_3\sqrt{\frac{g}{l}} = 3\frac{\pi}{2} \mid$

$t_3 = 3\frac{\pi}{2}\sqrt{\frac{l}{g}}$

$\varphi = 0 \mid t_4\sqrt{\frac{g}{l}} = 2\pi$

$t_4 = 2\pi\sqrt{\frac{l}{g}}$

t_4 = Periode $= 2\pi\sqrt{\frac{l}{g}}$

[illegible]

[illegible]

[illegible]

Genaue Pendelformel

$\varphi \quad t$

$$\frac{d\varphi}{\sqrt{\cos\varphi - \cos\varphi_1}} = dt\sqrt{\frac{2g}{l}}$$

[illegible] $\cos\varphi > \cos\varphi_1$ [illegible]

[illegible] $\varphi = \varphi_1$

$$\cos\varphi = 1 - 2\sin^2\frac{\varphi}{2}$$

$$\cos\varphi_1 = 1 - 2\sin^2\frac{\varphi_1}{2}$$

$$\cos\varphi - \cos\varphi_1 = 2\left(\sin^2\frac{\varphi_1}{2} - \sin^2\frac{\varphi}{2}\right)$$

$$\sin\frac{\varphi}{2} = \sin\frac{\varphi_1}{2}\cdot\sin u$$ [[illegible] 1]

$$\cos\varphi - \cos\varphi_1 = 2\sin^2\frac{\varphi_1}{2}\left(1 - \sin^2 u\right)$$

$$\sqrt{\cos\varphi - \cos\varphi_1} = \sqrt{2}\,\sin\frac{\varphi_1}{2}\cos u$$

$$\cos\frac{\varphi}{2}\,d\frac{\varphi}{2} = \sin\frac{\varphi_1}{2}\cos u\,du$$

$$d\varphi = \frac{2\sin\frac{\varphi_1}{2}\cos u\,du}{\sqrt{1 - \sin^2\frac{\varphi_1}{2}\sin^2 u}}$$

$$\frac{2\sin\frac{\varphi_1}{2}\cos u\,du}{\sqrt{1-\sin^2\frac{\varphi_1}{2}\sin^2 u}\;\sqrt{2}\sin\frac{\varphi_1}{2}\cos u} = dt\sqrt{\frac{2g}{l}}$$

$$\frac{du}{\sqrt{1-\sin^2\frac{\varphi_1}{2}\sin^2 u}} = dt\sqrt{\frac{g}{l}}$$

[illegible] 180°

[illegible] = 0

$$\int_0^{\frac{\pi}{2}}\frac{du}{\sqrt{1-\sin^2\frac{\varphi_1}{2}\sin^2 u}} = \sqrt{\frac{g}{l}}\int_0^{t_1} dt$$

$$\left(1-\sin^2\frac{\varphi_1}{2}\sin^2 u\right)^{-\frac{1}{2}} = 1+\frac{1}{2}\sin^2\frac{\varphi_1}{2}\sin^2 u+\frac{3}{8}\sin^4\frac{\varphi_1}{2}\sin^4 u$$

$$- - - -$$

$$\int_0^{\frac{\pi}{2}}\left(1+\frac{1}{2}\sin^2\frac{\varphi_1}{2}\sin^2 u+\frac{3}{8}\sin^4\frac{\varphi_1}{2}\sin^4 u - \dots\right)du = t_1\sqrt{\frac{g}{l}}$$

$$\frac{\pi}{2} = t_1\sqrt{\frac{g}{l}}$$

$$t_1 = \frac{\pi}{2}\sqrt{\frac{l}{g}}$$ [illegible]

$\int_0^{\frac{\pi}{2}}\sin^2 u = i$ [illegible]

$= \int_0^{\frac{\pi}{2}}\cos^2 u = i$ $\quad 2i = \int_0^{\frac{\pi}{2}} du = \frac{\pi}{2}$ $\quad i = \frac{\pi}{4}$

$\frac{\pi}{2} + \frac{1}{2}\sin^2\frac{\varphi_1}{2}\,\frac{\pi}{4} = t_1\sqrt{\frac{g}{l}}$

$\frac{\pi}{2}\sqrt{\frac{l}{g}}\left[1+\frac{1}{4}\sin^2\frac{\varphi_1}{2}\right] = t_1$

[illegible] [...]

[illegible] t_1 [illegible]

[illegible]

[illegible] $\frac{1}{8}$ [illegible]

[illegible]

[illegible]

[illegible] $\sin 5^\circ = 0{,}087$

696

609

$0{,}007569 : 4 = 0{,}00189$

[illegible] $\frac{19}{10.000}$ [illegible] $\frac{2}{1000}$ [illegible]

[illegible] $\varphi_1 = 2^\circ 20'$ [illegible]

$\sin 1^\circ 10' = 0{,}02$

$0{,}0004$

$0{,}0001$ [illegible]

[illegible]

$$\underbrace{\int_0^{\frac{\pi}{2}} \sin^n u \, du}_{J^n} = \underbrace{\int_0^{\frac{\pi}{2}} \sin^{n-2} u \, du}_{J^{n-2}} - \underbrace{\int_0^{\frac{\pi}{2}} \sin^{n-2} u \cos^2 u \, du}$$

$$J^n = J^{n-2} - \int_0^{\frac{\pi}{2}} \cos u \, \frac{d \sin^{n-1} u}{du \,(n-1)}$$

Int. per partes

$$J^n = J^{n-2} - \left. \cos u \, \frac{\sin^{n-1} u}{n-1} \right|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\sin^{n-1} u}{n-1} \sin u \, du$$

$n > 1$

$$J^n = J^{n-2} - \frac{J^n}{n-1}$$

$$J^n = \frac{n-1}{n} J^{n-2}$$

Recursionsformel

$n = 2$

$$J^2 = \frac{1}{2} J^0 = \frac{\pi}{4}$$

$$J^4 = \frac{3}{4} \cdot \frac{\pi}{4} = \frac{3\pi}{16}$$

[... ... $- \frac{[illegible]}{64} \sin^4 \frac{\pi}{2}$]

[illegible] 180° [illegible]

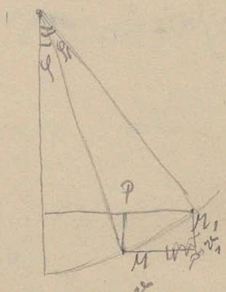
[illegible]

[illegible]

~~[illegible]~~ [illegible] $\frac{\pi}{2}$ [illegible]

$\arcsin \frac{\varphi_2}{\varphi_1}$

[illegible]



[illegible]

[illegible]

$m\frac{v^2}{2}$ $\quad$ $m\frac{v_1^2}{2}$ $\quad$ [illegible]

$m\frac{v^2}{2} - m\frac{v_1^2}{2} =$ [illegible] $=$ [illegible]

$= mg\,MP$

$MP = l\cos\varphi - l\cos\varphi_1$

$$v^2 - v_1^2 = 2gl(\cos\varphi - \cos\varphi_1)$$

[illegible]

$$v_1 = 0$$

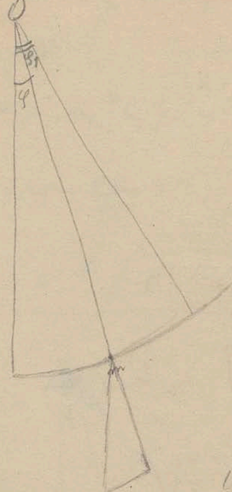
$$v^2 = 2gl(\cos\varphi - \cos\varphi_1)$$

$$l^2\left(\frac{d\varphi}{dt}\right)^2 =$$

$$\left(\frac{d\varphi}{dt}\right)^2 = \frac{2g}{l}\sqrt{[\cos\varphi - \cos\varphi_1]}$$ [illegible]

[illegible]

9/1



$\frac{mv^2}{\rho} = |$

[illegible]

$-mg\cos\varphi$

$s = \frac{mv^2}{r} + mg\cos\varphi$ [illegible]

[illegible]

[illegible]

[illegible] $\frac{mv^2}{r}$ [illegible]

[illegible] $\frac{mv^2}{r}$ — Centrifugal [illegible]

[illegible] $s = mg$

[illegible] $= mg + \frac{mv^2}{r}$

[illegible] O : [illegible]

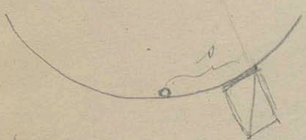
[illegible]

$\frac{mv^2}{2} = mgl[\cos\varphi - \cos\varphi_1]$

$v^2 = 2gl[\cos\varphi - \cos\varphi_1]$

$s = \frac{2mgl[\cos\varphi - \cos\varphi_1]}{l} + mg\cos\varphi$

$\lambda = mg\left[3\cos\varphi - 2\cos\varphi_1\right]$ [illegible]



[illegible]

$m\dfrac{d^2s}{dt^2} = -mg\sin\varphi$

[illegible] $= N$

$N = \dfrac{mv^2}{r} - mg\cos\varphi$

$N = \dfrac{mv^2}{r} + mg\cos\varphi$

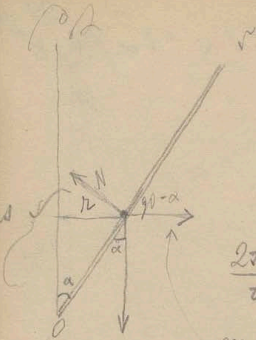
[illegible]

$\lambda = mg\left[3\cos\varphi - 2\cos\varphi_1\right]$ [illegible]

[illegible]

[illegible]

[illegible]



[illegible]

[illegible]

$\tau =$

$$\frac{2\pi r}{\tau} = u$$

$$\frac{mu^2}{r}$$

$$m\frac{d^2s}{dt^2} = \frac{mu^2}{r}\cos(90-\alpha) - mg\cos\alpha$$

$$= \frac{mu^2}{r}\sin\alpha - mg\cos\alpha$$

$$m\frac{d^2s}{dt^2} = \frac{m}{r}\,\frac{4\pi^2r^2}{\tau^2}\sin\alpha - mg\cos\alpha$$

$$r = s\sin\alpha$$

$$\frac{d^2s}{dt^2} = \frac{4\pi^2\sin^2\alpha}{\tau^2}\cdot s - g\cos\alpha$$

[illegible]

[illegible] $\frac{d^2s}{dt^2}$ [illegible]

[illegible] $\frac{4\pi^2\sin^2\alpha}{\tau^2}s_0 - g\cos\alpha = 0$ | [illegible]

[illegible] τ [illegible] s_0 [illegible]

[illegible]

[illegible] $\frac{dx}{dt}$ multipliert

[illegible]:

[illegible]:

$$s = x + a \qquad a = \text{Const.}$$

$$\frac{d^2 s}{dt^2} = \frac{d^2 x}{dt^2}$$

$$\frac{d^2 x}{dt^2} = \frac{4\pi^2 \sin^2\alpha}{\tau^2}\, x + \underbrace{\frac{4\pi^2 \sin^2\alpha}{\tau^2}\, a - g\cos\alpha}$$

[illegible]

$$\frac{4\pi^2 \sin^2\alpha}{\tau^2}\, a = g\cos\alpha$$

$$\left.\frac{4\pi^2 \sin^2\alpha}{\tau^2}\right\} = \lambda^2$$

$$\frac{d^2 x}{dt^2} = \lambda^2 x \qquad x = f(t)$$ [illegible] 2te diff. [illegible] Const. [illegible]

$$x = e^{\mu t}$$

$$\frac{dx}{dt} = \mu e^{\mu t}$$

$$\frac{d^2 x}{dt^2} = \mu^2 e^{\mu t}$$

$$= \mu^2 x$$

$$\mu^2 = \lambda^2 \quad [\text{illegible}] \quad x = \begin{cases} e^{\lambda t} \\ e^{-\lambda t} \end{cases} ; \quad \begin{matrix} A e^{\lambda t} \\ B e^{-\lambda t} \end{matrix}$$ [illegible]

$$\mu = \pm\lambda$$

$$x = A e^{\lambda t} + B e^{-\lambda t}$$

$$s = A e^{\lambda t} + B e^{-\lambda t} + \frac{g\cos\alpha}{\lambda^2}$$

$A, B = ?$

$t=0 \quad |s=0| \quad \frac{ds}{dt}=0$

$s_0 = A + B + \frac{g\cos\alpha}{\lambda^2}$

$\frac{ds}{dt} = 0 = \lambda A - \lambda B$

$A = B$

$s_0 - \frac{g\cos\alpha}{\lambda^2} = 2A$

$s = \frac{1}{2}\left[s_0 - \frac{g\cos\alpha}{\lambda^2}\right]\left[e^{\lambda t} + e^{-\lambda t}\right] + \frac{g\cos\alpha}{\lambda^2}$

[illegible] + [illegible] $t = \infty$ $s = \infty$

— $= 0$ — — [illegible] $s = \frac{g\cos\alpha}{\lambda^2}$

— — — — [illegible]

[illegible]

[illegible]

$$l\frac{d^2\varphi}{dt^2} = -g\sin\varphi$$

$$\frac{d^2\varphi}{dt^2} = -\frac{g}{l}\cdot\varphi$$

[illegible]

[illegible]

Exponentialf.

$$\varphi = e^{\alpha t}$$

$$\frac{d\varphi}{dt} = \alpha e^{\alpha t}$$

$$\frac{d^2\varphi}{dt^2} = \alpha^2 e^{\alpha t}$$

$$\alpha^2 e^{\alpha t} = -\frac{g}{l} e^{\alpha t}$$

$$\alpha^2 = -\frac{g}{l}$$

$$\alpha = \pm i\sqrt{\frac{g}{l}}$$

[illegible] imag. [illegible] ± Exp. f. imag. Expon.

$$\alpha_1 \;\text{---}\; +i\sqrt{\ }$$

$$\alpha_2 \;\text{---}\; -i\sqrt{\ }$$

$$\varphi = e^{\alpha_1 t} \qquad \varphi = e^{\alpha_2 t}$$

[illegible]

$$\varphi = Ae^{\alpha t}$$

[illegible] $A_1 e^{\alpha_1 t}$, $A_2 e^{\alpha_2 t}$ } [illegible]

[illegible] $: A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$

[illegible] linear [illegible]

$$\varphi = A_1 e^{it\sqrt{\frac{g}{l}}} + A_2 e^{-it\sqrt{\frac{g}{l}}}$$

$$\varphi = \underbrace{(A_1 + A_2)}\cos t\sqrt{\tfrac{g}{l}} + \underbrace{(A_1 i - A_2 i)}\sin t\sqrt{\tfrac{g}{l}}$$

$$= C \cos t\sqrt{\tfrac{g}{l}} + D \sin t\sqrt{\tfrac{g}{l}}$$

[illegible] Constanten [illegible]

t_1 = Periode

$t_1\sqrt{\frac{g}{l}} = 2\pi$ [illegible]

$t_1 = 2\pi\sqrt{\frac{l}{g}}$ = [illegible]

[illegible]



[illegible]

$$m\frac{d^2s}{dt^2} = -mg\sin\varphi - \beta\frac{ds}{dt}$$

$-\beta\frac{ds}{dt}$ [illegible]

$$s = l\varphi$$

$\sin\varphi = \varphi$ [illegible]

$$\frac{d^2\varphi}{dt^2} = -\underbrace{\frac{g}{l}}_{a^2}\varphi - \underbrace{\frac{\beta}{m}}_{2b}\frac{d\varphi}{dt}$$

$$\frac{d^2\varphi}{dt^2} + 2b\frac{d\varphi}{dt} + a^2\varphi = 0$$

$$\varphi = Ae^{\alpha t}$$

[illegible]

$\frac{d\varphi}{dt} = \alpha Ae^{\alpha t}$ $\qquad$ $\frac{d^2\varphi}{dt^2} = \alpha^2 Ae^{\alpha t}$

$$Ae^{\alpha t}\left[\alpha^2 + 2b\alpha + a^2\right] = 0$$

[illegible]

$$\alpha^2 + 2b\alpha + a^2 = 0$$

$$\alpha = -b \pm \sqrt{b^2 - a^2}$$

$$\alpha_1 = -b + \sqrt{}$$

$$\alpha_2 = -b - \sqrt{}$$

$A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} = \varphi$ [illegible]

$b \lessgtr a^2$ [illegible] e^{bt}

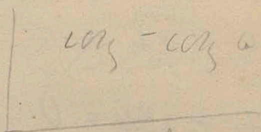
1.) $\sqrt{b^2 - a^2}$

[illegible] – Exp.

[illegible] $t = \infty$ $\varphi = 0$

2.) $b^2 > a^2$ [illegible] α_1, α_2 [illegible] neg. [illegible]

$b^2 < a^2$ [illegible]



[illegible] neg.

$\alpha_1 = -b + i\sqrt{a^2 - b^2}$

$= -b + ic$

$\alpha_2 = -b - ic$

$\varphi = e^{-bt}\left[A_1 e^{cti} + A_2 e^{-cti}\right]$

$\varphi = e^{-bt}\left[M \cos ct + N \sin ct\right]$

$M = A_1 + A_2 \qquad N = A_1 i - A_2 i$

[illegible] $z = 0$

< e periodi [illegible]

periodi [illegible]

[illegible] e Const. M, N

$t = 0$

$$\varphi = e^{-bt}\left[M\cos ct + N\sin ct\right] \qquad \text{I}$$

$$\left.\begin{aligned}\frac{d\varphi}{dt} &= -b\,e^{-bt}\left[M\cos ct + N\sin ct\right] + e^{-bt}\Big[\\ &\left[-c\,M\sin ct + c\,N\cos ct\right]\end{aligned}\right\} \text{II}$$

$t = 0 \quad \varphi = \varphi_1$ [[illegible] $t = 0$]

[illegible]

$$\frac{d\varphi}{dt} = 0$$

I $\varphi_1 = M$

II $0 = -bM + cN$

$$N = \frac{b}{c} M$$

$$\varphi = \varphi_1 e^{-bt}\left[\cos ct + \frac{b}{c}\sin ct\right]$$

$$\frac{d\varphi}{dt} = e^{-bt}\Big[\cos ct\underbrace{(-bM + cN)}_{=0} + \sin ct(-bN - cM)\Big]$$

$$\frac{d\varphi}{dt} = e^{-bt}\sin ct\left(-\frac{b^2}{c}M - cM\right)$$

$$\frac{d\varphi}{dt} = -\varphi_1 \frac{b^2+c^2}{c} e^{-bt} \sin ct$$

[illegible] φ

[illegible]

$\dot\varphi = 0$, $t = 0$ [illegible]

[illegible] $ct_1 = \pi$

$t_1 = \frac{\pi}{c}$

[illegible] $ct_2 = 2\pi$

$t_2 = \frac{2\pi}{c}$

[illegible]

[illegible]

[illegible] $t = 0$ $\varphi = \varphi_1$

$ct_1 = \pi$ $\varphi = -\varphi_1 e^{-bt_1}$

$c t_2 = 2\pi$

$\varphi = +\varphi_1 e^{-b t_2}$

$c t_3 = 3\pi$

$\varphi = -\varphi_1 e^{-b t_3}$

[illegible]

[illegible]

[illegible] $\tau = \frac{\pi}{c}$

$c = \sqrt{a^2 - b^2}$

$\tau = \frac{\pi}{\sqrt{a^2 - b^2}}$

[illegible]

$t = \frac{\pi}{a} = \pi \sqrt{\frac{l}{g}}$

[illegible]

[illegible] $\tau = \frac{\pi}{a\sqrt{1 - \frac{b^2}{a^2}}} = \frac{\pi}{a}\left[1 + \frac{1}{2}\frac{b^2}{a^2} + \cdots\right]$

[illegible]

[illegible]

[illegible]

[illegible] $b = a$ [illegible] $T = \infty$

[illegible] — aperiodisch [illegible]

[illegible] $b > a$ [illegible] transf. [illegible]

[illegible] perio[illegible]

[illegible] Galvanometer [illegible]

[illegible]

[illegible]

$\varphi_0 \qquad (-)\varphi_1 e^{-bt_1} \qquad + \varphi_2 e^{-bt_2}$

$\uparrow \qquad \uparrow$

$x_1 \qquad x_2$

$T = t_1$

$$\frac{x_1}{x_2} = e^{bt_1} \qquad bt = l\frac{x_1}{x_2}$$

[illegible]

[illegible]

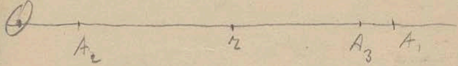
[illegible]

[illegible] 2 & 3 [illegible] 10 Minuten

[illegible]

[illegible]

14/1 [illegible] [Sakrament [illegible]



[illegible]

$rA_1 = \varphi_1$

$rA_2 = \varphi_2$

$rA_3 = \varphi_3$

$\varphi_2 = -\varphi_1 e^{-b\tau}$

$\varphi_3 = +\varphi_1 e^{-2b\tau}$

[illegible] O = [illegible] Scala

$rA_1 = OA_1 - Or$

$rA_2 = Or - OA_2$

$rA_3 = OA_3 - Or$

[illegible]

$\varphi_2 = Or - OA_2 = (OA_1 - Or)e^{-b\tau}$

$\varphi_3 = OA_3 - Or = (OA_1 - Or)e^{-2b\tau}$

$$\frac{(Or - OA_2)^2}{OA_3 - Or} = \frac{(OA_1 - Or)^2}{OA_1 - Or}$$

$$(Or - OA_2)^2 = (OA_1 - Or)(OA_3 - Or)$$

$$Or^2 - 2\,Or\,OA_2 + OA_2^2 = OA_1\,OA_3 - Or(OA_1 + OA_3) + Or^2$$

$$O_r = \frac{OA_1 \, OA_3 - OA_2^2}{OA_1 + OA_3 - 2OA_2}$$

[illegible]

[illegible]

$$O_r - OA_2 = (OA_1 - O_r)(1 - b\tau)$$
$$OA_3 - O_r = (OA_1 - O_r)(1 - 2b\tau)$$
} [illegible]

$$2O_r - OA_2 - OA_1 = -(OA_1 - O_r)\,b\tau \quad \Big| \; 2$$
$$OA_3 - OA_1 = -(OA_1 - O_r)\,2b\tau$$

$$4O_r - 2OA_2 - 2OA_1 - OA_3 + OA_1 = 0$$

$$O_r = \frac{OA_1 + OA_3 + 2OA_2}{4}$$

[illegible] arith. [illegible]

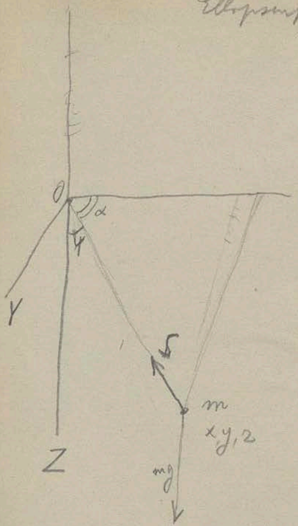
[illegible] neg. [illegible]

[[illegible]]

[illegible]

[illegible] [[illegible]]

Ellipsenpendel



[illegible]

$-S\cos\alpha$

$x = l\cos\alpha$

$\cos\alpha = \frac{x}{l}$

$-S\frac{x}{l}$

$$m\frac{d^2x}{dt^2} = -S\frac{x}{l} \qquad y$$

$$m\frac{d^2y}{dt^2} = -S\frac{y}{l} \qquad x$$

$$m\frac{d^2z}{dt^2} = mg - S\frac{z}{l}$$

$$x^2+y^2+z^2 = l^2$$

[illegible]

[illegible]

[illegible]

[illegible]

$$m\left[y\frac{d^2x}{dt^2} - x\frac{d^2y}{dt^2}\right] = 0$$

[illegible] const. [illegible] Centralb. [illegible]

[illegible]

Kreisbahn

[illegible]

[illegible]

$z = \text{constant}$

$z = l \cos\psi$ [ψ const]

$\frac{d^2 z}{dt^2} = 0$

$0 = mg - S\frac{z}{l}$

$S = \frac{mg}{\cos\psi}$

[illegible]

$S = \frac{mg}{\cos\psi}$

[illegible] v [const.], τ

$v = \frac{2\pi r}{\tau} = \frac{2\pi\, l \sin\psi}{\tau}$

[illegible]



[illegible]: $\frac{mv^2}{R} = S\sin\psi$

$\frac{mv^2}{r} = mg\frac{\sin\psi}{\cos\psi}$

$v^2 = lg\frac{\sin^2\psi}{\cos\psi}$

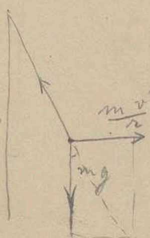
$\frac{4\pi^2 l^2 \sin^2\psi}{\tau^2} = lg\frac{\sin^2\psi}{\cos\psi}$ ‖ $\frac{4\pi^2 l}{g}\cos\psi = \tau^2$

$$\tau = 2\pi\sqrt{\frac{l\cos\varphi}{g}}$$

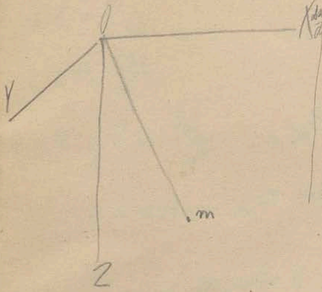
[illegible] $\tau = 2\pi\sqrt{\frac{l}{g}}$

[illegible]

[illegible]: [illegible]



$$\frac{mv^2}{r}\cos\varphi = mg\sin\varphi$$



$X \frac{dx}{dt}$

x	$m\frac{d^2x}{dt^2} = -S\frac{x}{l}$
y	$m\frac{d^2y}{dt^2} = -S\frac{y}{l}$
z	$m\frac{d^2z}{dt^2} = mg - S\frac{z}{l}$

$x^2+y^2+z^2 = l^2$

$$m\left[x\frac{d^2x}{dt^2} + \sim\right] = mgz - Sl$$

$$S = \frac{mgz}{l} - m\left[\frac{x}{l}\frac{d^2x}{dt^2} + \frac{y}{l}\frac{d^2y}{dt^2} + \frac{z}{l}\frac{d^2z}{dt^2}\right]$$

[illegible]

[illegible]

[illegible]

$$m\left[\frac{dx}{dt}\frac{d^2x}{dt^2} + d \cdots + \right] = mg\frac{dz}{dt} -$$

$$-S\left[x\frac{dx}{dt} + \cdots\right]$$

$x\frac{dx}{dt} +$ [illegible] $= 0$

$S = 0$

[illegible]

[illegible]:

[illegible] $x\,y$ [illegible]

$\sin\varphi = \varphi$

$\cos\varphi = 1$ [[illegible]]

[illegible]

$z = \sqrt{l^2 - x^2 - y^2}$ [illegible]

$S = mg$

$$m\frac{d^2x}{dt^2} = -mg\frac{x}{l}$$

$$\frac{d^2x}{dt^2} = -\frac{g}{l}\,x$$

$$\frac{d^2y}{dt^2} = -\frac{g}{l}\,y$$

[illegible]

$$\frac{d^2x}{dt^2} = -a^2x \qquad a^2 = \frac{g}{l}$$

$$\frac{d^2y}{dt^2} = -a^2y$$

$$x = A\cos at + B\sin at$$

$$y = C\cos at + D\sin at$$

[illegible]

Ellipse

[illegible]

$at = 2\pi$

$\tau = \frac{2\pi}{a}$

$\tau = 2\pi \sqrt{\frac{l}{g}}$

$\frac{dx}{dt} = -aA \sin at + aB \cos at$

$\frac{dy}{dt} = -aC \sin at + aD \cos at$

[illegible] [illegible RZZ]

$t=0 \quad x=p$ [illegible]

$y=0$

$t=0 \quad \frac{dx}{dt}=0$

$\frac{dy}{dt} = q$

$p = A \quad 0 = C \quad 0 = aB$

$q = aD$

$x = p \cos at \qquad \frac{x}{p} = \cos at \qquad \frac{x^2}{p^2} + \frac{a^2y^2}{q^2} = 1$

$y = \frac{q}{a} \sin at \qquad \frac{ay}{q} = \sin at \qquad$ [illegible] Ell.

[illegible] $p = \frac{q}{a} \quad q = ap$ [illegible] Ell.

[illegible]

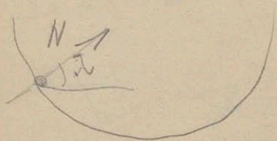
[illegible]

[illegible]

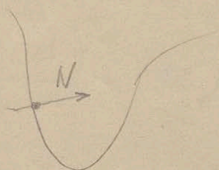
[illegible]

[illegible]

$$m\frac{d^2x}{dt^2} = X + N \cos\lambda$$



[illegible]



$f(x,y,z)=0$ [illegible]

$$\cos\lambda = \frac{\frac{df}{dx}}{\pm\sqrt{\left(\frac{df}{dx}\right)^2+\left(\frac{df}{dy}\right)^2+\left(\frac{df}{dz}\right)^2}}$$

[illegible]

[illegible]

[illegible]

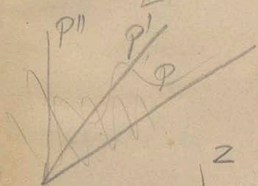
[illegible]

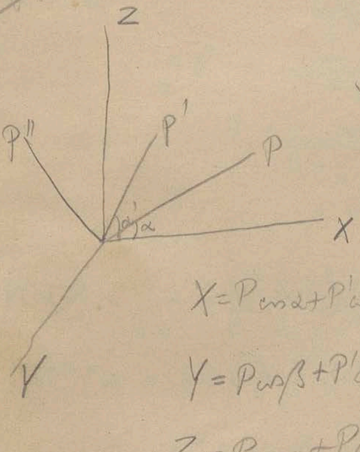
[illegible]

[illegible]

[illegible]

[illegible] R [illegible] $\angle = 0$ [illegible]





$P\cos\alpha + P'\cos\alpha'$

$X =$

$X = P\cos\alpha + P'\cos\alpha' + \dots$

$Y = P\cos\beta + P'\cos\beta' + \dots$

$Z = P\cos\gamma + P'\cos\gamma' + \dots$

[illegible]

$R^2 = X^2 + Y^2 + Z^2$ [illegible] R [illegible] 0 [illegible]

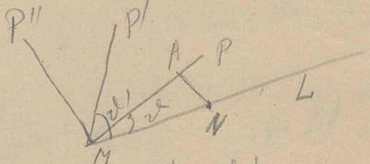
[illegible] $= 0$ [illegible]

$X=0$ [illegible] Coord. [illegible]

$Y=0$ [illegible]

$Z=0$

[illegible]



$P\cos\vartheta \quad P'\cos\vartheta'$

$L = P\cos\vartheta + P'\cos\vartheta' - \quad =0$

$\underbrace{P\cos\vartheta\, MN} + \underbrace{P'\cos\vartheta' MN} + \quad = 0$

[illegible] P

[illegible] MN [illegible]

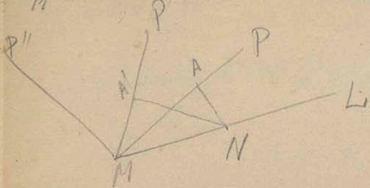
[illegible]

[illegible]

[illegible] $=0$ [illegible]

$MN\cos\vartheta = MA$

$P\, MA + P'\, MA' + \quad =0$



[illegible]

$PMA + P'MA' + \ldots = 0$

[illegible]

$MA = \delta p$

$MA' = \delta p'$

$P\,\delta p + P'\,\delta p' = \ldots = 0$

$\sum P\,\delta p = 0$ virtuelle [illegible]

δl [illegible]

[illegible]

$X\ Y\ Z$ δs [illegible]

$\delta x\ \delta y\ \delta z$ [illegible]

$X\,\delta x + Y\,\delta y + Z\,\delta z = 0$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible] $X = 0$

[illegible] $Y = 0$ } [illegible] $R = 0$ [illegible]

$Z = 0$ [illegible] $= 0$

[illegible]

$F(x, y, z) = 0$

$F(x+\delta x, y+\delta y, z+\delta z) = 0$ [illegible]

[illegible]

[illegible] Taylor'schen [illegible]

$$\underbrace{F(x,y,z)}_{=0} + \delta x \frac{dF}{dx} + \delta y \frac{dF}{dy} + \delta z \frac{dF}{dz} \text{ etc.} = 0$$

$$\delta x \frac{dF}{dx} + \delta y \frac{dF}{dy} + \delta z \frac{dF}{dz} = 0$$

$$\delta x \frac{\frac{dF}{dx}}{\sqrt{\ldots}} + \ldots$$

$$\frac{\delta x}{\delta s} \frac{\frac{dF}{dx}}{\sqrt{\ldots}}$$

$\frac{\delta x}{\delta s} = \cos$ [illegible]

$\cos\alpha \cos\lambda + \cos\beta\cos\mu + \cos\gamma\cos\nu = 0$

$\left.\begin{matrix}\lambda\\ \mu\\ \nu\end{matrix}\right\}$ [illegible]

$\left.\begin{matrix}\alpha\\ \beta\\ \gamma\end{matrix}\right\}$ [illegible]

[illegible]

[illegible]

$X\delta x + Y\delta y + Z\delta z = 0$ [illegible]

$-\delta z$ [illegible]

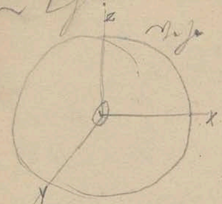
$$\delta z = -\frac{\delta x \frac{dF}{dx} + \delta y \frac{dF}{dy}}{\frac{dF}{dz}}$$

$$\left(X - \frac{Z\frac{dF}{dx}}{\frac{dF}{dz}}\right)\delta x + \left(Y - \frac{Z\frac{dF}{dy}}{\frac{dF}{dz}}\right)\delta y = 0$$

[illegible]

$$X - \frac{Z\frac{dF}{dx}}{\frac{dF}{dz}} = 0 \quad \Bigg| \quad Y - \frac{Z\frac{dF}{dy}}{\frac{dF}{dz}} = 0$$

[illegible]



$r=a$

$X=0 \mid Y=0 \mid Z=mg$

$x^2+y^2+z^2=a^2$

$F(x,y,z)=0$

$x^2+y^2+z^2-a^2=0$

$\frac{dF}{dx}=2x \quad \frac{dF}{dy}=2y \quad \frac{dF}{dz}=2z$

$-mg\,\frac{\frac{dF}{dx}}{\frac{dF}{dz}}=0 \quad \Big| \quad \frac{2x}{2z}=0 \quad \frac{x}{z}=0$

$\frac{y}{z}=0$

[illegible] $y=a$ [illegible] $x=0$, $y=0$

[illegible]

[illegible] dz

$\frac{X}{\frac{dF}{dx}}=\frac{Z}{\frac{dF}{dz}}=\frac{Y}{\frac{dF}{dy}}\left(=\frac{Z}{\frac{dF}{dy}}\right)=-\lambda$

[illegible]

~~$X = -\lambda$~~

$$X + \lambda \frac{dF}{dx} = 0$$

$$Y + \lambda \frac{dF}{dy} = 0$$

$$Z + \lambda \frac{dF}{dz} = 0$$

[illegible]

[illegible]

$$X \delta x + Y \delta y + Z \delta z = 0$$

$$\frac{dF}{dx} \delta x + \frac{dF}{dy} \delta y + \frac{dF}{dz} \delta z = 0 \cdot \lambda$$ [illegible]

$$\left(X + \lambda \frac{dF}{dx}\right) \delta x + \left(Y + \lambda \frac{dF}{dy}\right) \delta y + \left(Z + \lambda \frac{dF}{dz}\right) \delta z = 0$$

[illegible] δx δy

[illegible]

[illegible]

$$Z + \lambda \frac{dF}{dz} = 0$$

[illegible] δx [illegible] δy [illegible]

$$X + \lambda \frac{dF}{dx} = 0$$

$$Y + \lambda \frac{dF}{dy} = 0$$

[illegible] :

$X\delta x + Y\delta y + Z\delta z = 0$

[illegible]

[[illegible] X [illegible] $X\delta x = 0$ [illegible]]

$\varphi(x, y, z) = 0$

$\psi(x, y, z) = 0$

$\varphi(x+\delta x,\ y+\delta y,\ z+\delta z) = 0$

$$\left.\begin{array}{l} \underset{=0}{\varphi} + \frac{d\varphi}{dx}\delta x + \frac{d\varphi}{dy}\delta y + \frac{d\varphi}{dz}\delta z = 0 \\ \underset{=0}{\psi} + \frac{d\psi}{dx}\delta x + \frac{d\psi}{dy}\delta y + \frac{d\psi}{dz}\delta z = 0 \end{array}\right\} \begin{array}{l} \lambda \\ \mu \end{array}$$

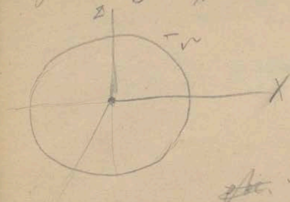
$$\left(X + \lambda\frac{d\varphi}{dx} + \mu\frac{d\psi}{dx}\right)\delta x + \left(Y + \ldots\right)\delta y + (\ \ )\delta z = 0$$

λ, μ [illegible] δz, δy [illegible]

[illegible] ; δx [illegible] $= 0$ [illegible] :

$$\left\{\begin{array}{l} X + \lambda\frac{d\varphi}{dx} + \mu\frac{d\psi}{dx} = 0 \\ Y + \ldots \\ Z + \ldots \end{array}\right\}$$

[illegible]



$y = 0$

$x^2 + z^2 = a^2$

~~$X + \lambda . 0$~~

$0 + \lambda . 0 + \mu 2x = 0$

$0 + \lambda . 1 + \mu . 0 = 0$

$-mg + \lambda 0 + \mu . 2z = 0$

$\mu 2x = 0$

$-mg + \mu 2z = 0$

[illegible]

$z = \pm a$

$\mu = \pm \frac{mg}{2a}$

[illegible]

[illegible]

[illegible]

$X\,\delta x + Y\,\delta y + Z\,\delta z \geqq 0$ [illegible]

[illegible]

$\therefore$

$x^2+y^2+z^2-a^2=0$

or $x^2+y^2+z^2-a^2 \geqq 0$ } [illegible]

$l \sim l \qquad x^2+y^2+z^2 \leqq l^2$

[illegible] d'Alembert

[illegible] d'Alembert [illegible]

[illegible]

[illegible] [Res.] [illegible]

Compon. Res. [illegible]

[illegible]

$X \mid Y \mid Z$ } 3 Compon. [illegible]

$-\left(m\frac{d^2x}{dt^2}\right) \mid -\left(m\frac{d^2y}{dt^2}\right) \mid -\left(m\frac{d^2z}{dt^2}\right)$ } [illegible]

[illegible]

$$\left(X - m\frac{d^2x}{dt^2}\right)\delta x + \left(Y - m\frac{d^2y}{dt^2}\right)\delta y + \left(Z - m\frac{d^2z}{dt^2}\right)\delta z = 0$$

[illegible] Dynamik [illegible]

[illegible]

$$X - m\frac{d^2x}{dt^2} = 0$$

[illegible]

$F(x,y,z)=0$

$$\frac{dF}{dx}\delta x + \frac{dF}{dy}\delta y + \frac{dF}{dz}\delta z = 0 \quad \Big| \lambda$$

[illegible]

$$\left[X - m\frac{d^2x}{dt^2} + \lambda\frac{dF}{dx}\right]\delta x + \left[\qquad\qquad\right]\delta y + \left[\quad\right]\delta z$$

[illegible]

[illegible] δx [illegible]

$= 0$

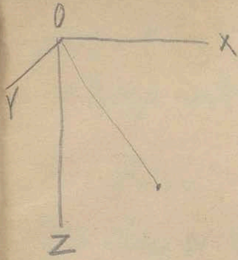
$$X - m\frac{d^2x}{dt^2} + \lambda\frac{dF}{dx} = 0$$

$$Y - m\frac{d^2y}{dt^2} + \lambda\frac{dF}{dy} = 0$$

$$Z - m\frac{d^2z}{dt^2} + \lambda\frac{dF}{dz} = 0$$

[illegible]

[illegible]



$$m\frac{d^2x}{dt^2} = X + \lambda \frac{dF}{dx}$$

$$F_{(xyz)} = x^2+y^2+z^2-l^2 = 0$$

$$\frac{dF}{dx} = 2x$$

$$\frac{dF}{dy} = 2y$$

$$\frac{dF}{dz} = 2z$$

$$m\frac{d^2x}{dt^2} = X + 2\lambda x$$

$$X = 0$$

$$Y = 0$$

$$Z = mg$$

$$m\frac{d^2x}{dt^2} = 2\lambda x$$

$$m\frac{d^2y}{dt^2} = 2\lambda y$$

$$m\frac{d^2z}{dt^2} = mg + 2\lambda z$$

$$x^2+y^2+z^2-l^2 = 0$$

$$2\lambda = -\frac{S}{l}$$

$$S = -2\lambda l$$

[illegible] -dynam. [illegible]

[illegible]

[illegible] dyn. [illegible]

[illegible] 2 Coord.

[illegible] 3. [illegible] 2 [illegible]; [illegible] 3. [illegible]

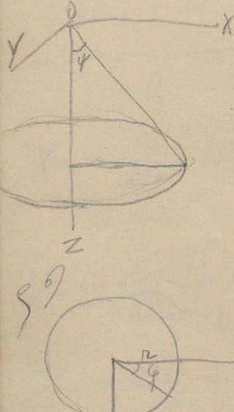
[illegible]

[illegible] Coord. [illegible]

[illegible]

[illegible]:

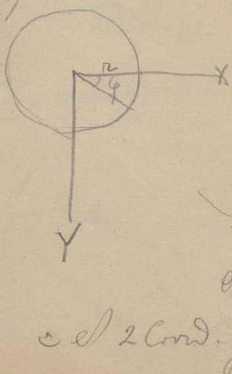
[illegible]:



[illegible] $x\ y\ z$

[illegible] l, ψ [[illegible]]

[illegible]



$$x = r\cos\varphi = l\sin\psi\cos\varphi$$
$$y = r\sin\varphi = l\sin\psi\sin\varphi$$
$$z = l\cos\psi$$

[illegible] 3 [illegible]

[illegible] 2 Coord. [illegible] ψ [illegible] φ

$X\,\delta x + Y\,\delta y + Z\,\delta z = 0$

[illegible]

$\delta x = l \cos\psi \cos\varphi \, \delta\psi - l \sin\psi \sin\varphi \, \delta\varphi$

$\delta y = l \cos\psi \sin\varphi \, \delta\psi + l \sin\psi \cos\varphi \, \delta\varphi$

$\delta z = - l \sin\psi \, \delta\psi$

$\delta\psi \left[X l \cos\psi \cos\varphi + \ldots\ldots \right] + \delta\varphi \left[X \ldots\ldots \right] = 0$

[illegible]

[illegible]

$X l \cos\psi \cos\varphi + Y l \cos\psi \sin\varphi - Z l \sin\psi = 0$

$X = 0 \qquad Y = 0 \qquad Z = mg$

[illegible]

$-mg\, l \sin\psi = 0$

$\sin\psi = 0$ [illegible]

[illegible]

$\frac{dx}{dt} = l \cos\psi \cos\varphi \frac{d\psi}{dt} - l \sin\psi \sin\varphi \frac{d\varphi}{dt}$

$$\frac{d^2x}{dt^2} = -l \sin\psi \cos\varphi \left(\frac{d\psi}{dt}\right)^2 - l \cos\psi \sin\varphi \frac{d\psi}{dt} \frac{d\varphi}{dt} +$$

$$+ l \cos\psi \cos\varphi \left(\frac{d^2\psi}{dt^2}\right)^2 + \ldots$$

3 [illegible]

[illegible]

[illegible]

[illegible]

$$X \delta x + Y \delta y + Z \delta z = 0$$

[illegible] U

$$X = \frac{dU}{dx} \qquad Y = \frac{dU}{dy} \qquad Z = \frac{dU}{dz}$$

$$\frac{dU}{dx} \delta x + \frac{dU}{dy} \delta y + \frac{dU}{dz} \delta z = 0$$

[illegible] δx [illegible]

δU

$\delta U = 0$ [illegible]

[illegible]

[[illegible]]

stabil labil

stabil

verschob.

21/1 Ableitung der Lagrange'schen Bewegungsgleichungen:

$$\left(X - m\frac{d^2x}{dt^2}\right)\delta x + \left(Y - m\frac{d^2y}{dt^2}\right)\delta y + \left(Z - m\frac{d^2z}{dt^2}\right)\delta z = 0$$

[illegible]

$$m\left(\frac{d^2x}{dt^2}\delta x + \frac{d^2y}{dt^2}\delta y + \frac{d^2z}{dt^2}\delta z\right) = X\delta x + Y\delta y + Z\delta z$$

[illegible]

p, q [illegible]

$x = f(p, q)$

$y = \varphi(p, q)$

$z = \psi(p, q)$

$$\delta x = \frac{df}{dp}\delta p + \frac{df}{dq}\delta q$$

$$\delta x = \frac{dx}{dp}\delta p + \frac{dx}{dq}\delta q$$

$$\delta y = \frac{dy}{dp}\delta p + \frac{dy}{dq}\delta q$$

$$\delta z = \frac{dz}{dp}\delta p + \frac{dz}{dq}\delta q$$

$$= \left[X\frac{dx}{dp} + Y\frac{dy}{dp} + Z\frac{dz}{dp}\right]\delta p + \left[\;\cdots\;\right]\delta q$$

[illegible]: $\cdots = 0$

[illegible] 2.

$$X\delta x + Y\delta y + Z\delta z = dU = \frac{dU}{dp}\delta p + \frac{dU}{dq}\delta q$$

$= 0$ [illegible]

$$\frac{dU}{dp} = 0 \qquad \frac{dU}{dq} = 0$$

[illegible] Coord.

pro $\rho \gamma \sigma$:

$\frac{dx}{dt} = x'$ [$\dot{x}$ v ~ ...]

$$\frac{dx'}{dt}\delta x + \frac{dy'}{dt}\delta y + \frac{dz'}{dt}\delta z$$

$$\left[\frac{dx'}{dt}\frac{dx}{dp} + \frac{dy'}{dt}\frac{dy}{dp} + \frac{dz'}{dt}\frac{dz}{dp}\right]\delta p + \left[\frac{dx'}{dt}\frac{dx}{dq} \ldots\right]\delta q = 0$$

$$\frac{dx'}{dt}\frac{dx}{dp} + \ldots + \ldots$$

$$\frac{d}{dt}\left(x'\frac{dx}{dp} + y'\frac{dy}{dp} + z'\frac{dz}{dp}\right) - x'\frac{d^2x}{dt\,dp} - y'\frac{d^2y}{dt\,dp}$$

$$-z'\frac{d^2z}{dt\,dp} =$$

$$= \frac{d}{dt}\left(\qquad\right) - x'\frac{dx'}{dp} - y'\frac{dy'}{dp} - z'\frac{dz'}{dp}$$

$$= \frac{d}{dt}\left(\qquad\right) - \frac{1}{2}\frac{d}{dp}\left[x'^2 + y'^2 + z'^2\right]$$

v ~ ... $v^2 = x'^2 + y'^2 + z'^2$

$$\frac{dx'}{dt}\frac{dx}{dp} + \frac{dy'}{dt}\frac{dy}{dp} + \frac{dz'}{dt}\frac{dz}{dp} = \frac{d}{dt}\left(x'\frac{dx}{dp} + y'\frac{dy}{dp} + z'\frac{dz}{dp}\right) -$$

$$- \frac{d}{dp}\left(\frac{v^2}{2}\right)$$

$x = f(p, q)$

$$\frac{dx}{dt} = \frac{df}{dp}\cdot\frac{dp}{dt} + \frac{df}{dq}\frac{dq}{dt} = \frac{dx}{dp}\frac{dp}{dt} + \frac{dx}{dq}\frac{dq}{dt}$$

$$x' = \frac{dx}{dp}p' + \frac{dx}{dq}q'$$

$$\frac{dx'}{dp'} = \frac{dx}{dp}$$

$$\text{---} = \frac{d}{dt}\left(x'\frac{dx'}{dp'} + y'\frac{dy'}{dp'} + z'\frac{dz'}{dp'}\right) - \frac{d}{dp}\left(\frac{v^2}{2}\right)$$

$$= \frac{d}{dt}\left[\frac{d}{dp'}\frac{1}{2}\left(x'^2 + y'^2 + z'^2\right)\right] - \frac{d}{dp}\left(\frac{v^2}{2}\right)$$

$$= \frac{d}{dt}\left[\frac{d}{dp'}\left(\frac{v^2}{2}\right)\right] - \frac{d}{dp}\left(\frac{v^2}{2}\right)$$

$$m\frac{v^2}{2} = L$$

$$\left[\frac{d}{dt}\left[\frac{dL}{dp'}\right] - \frac{dL}{dp}\right]\delta p + \left[\frac{d}{dt}\left[\frac{dL}{dq'}\right] - \frac{dL}{dq}\right]\delta q =$$

$$\frac{dU}{dp}\delta p + \frac{dU}{dq}\delta q$$

[illegible]

$$\frac{d}{dt}\left[\frac{dL}{dp'}\right] - \frac{dL}{dp} = \frac{dU}{dp}$$

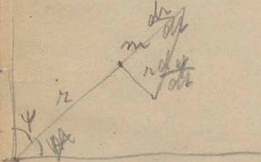
$= P$ [illegible]

$=$ [illegible] δ_p mult. $\frac{\sigma}{2}$ [illegible]

[illegible] $= P$

[illegible]:

[illegible] Polarcoord.



[illegible]:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\psi}{dt}\right)^2$$

$$L = m\frac{r'^2}{2} + m r^2 \frac{\psi'^2}{2}$$

$p = r$

$q = \psi$

$\frac{dL}{dr'} = m r'$ | $m\frac{dr'}{dt} - m r \psi'^2 = R$ [illegible]

$\frac{dL}{dr} = m r \psi'^2$ | $m\frac{d^2r}{dt^2} - m r\left(\frac{d\psi}{dt}\right)^2 = R$

$r\,\delta\psi$ $\quad S r \delta\psi$

[illegible] RV:

$\frac{dL}{d\psi'} = m r^2 \psi'$

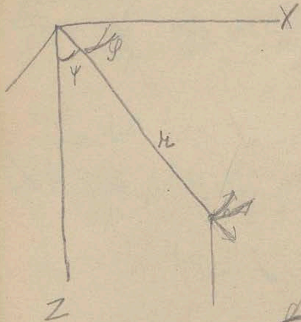
$\frac{dL}{d\psi} = 0$

$$\frac{d}{dt}\left(m r^2 \frac{d\psi}{dt}\right) = S r$$

[illegible]

[illegible]

[illegible] Lagrange [illegible] allg. Coord.



L

[illegible]

r, ψ, φ

$$\frac{dr}{dt} \quad \frac{r\,d\psi}{dt} \quad r \sin\psi \frac{d\varphi}{dt}$$

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\psi}{dt}\right)^2 + r^2 \sin^2\psi \left(\frac{d\varphi}{dt}\right)^2$$

$$L = \frac{m}{2}\left(r'^2 + r^2\psi'^2 + r^2 \sin^2\psi\, \varphi'^2\right)$$ [illegible]

$$\frac{dL}{d\varphi'} = m\, r^2 \sin^2\psi . \varphi'$$

$$\frac{dL}{d\varphi} = 0$$

[illegible]

$$\frac{d}{dt}\left(m\, r^2 \sin^2\psi \frac{d\varphi}{dt}\right) = 0$$

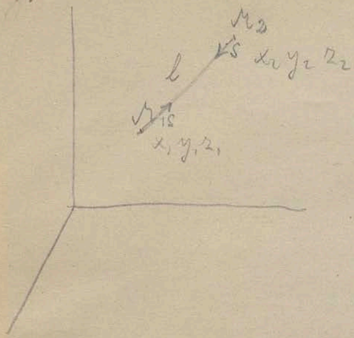
[illegible]

$$m\, r^2 \sin^2\psi \frac{d\varphi}{dt} = C$$

[illegible]

[illegible]

[illegible]



[illegible]

$X_1 Y_1 Z_1$

$X_1 \delta x_1 + Y_1 \delta y_1 + Z_1 \delta z_1 = 0$

[illegible]

$X_1 = 0 \quad Y_1 = 0 \quad Z_1 = 0$

[illegible]

[illegible]

$X_2 \delta x_2 + Y_2 \delta y_2 + Z_2 \delta z_2 = 0$

[illegible]

$X_1 \delta x_1 + X_2 \delta x_2 + Y_1 \delta y_1 + Y_2 \delta y_2 + Z \ldots\ldots = 0$

[illegible]

[illegible]

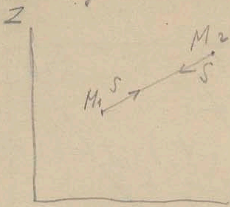
$(x_2 - x_1)^2 + (y_1 - y)^2 + (z_1 - z_2)^2 = l^2$

[illegible]

[illegible]

[illegible]

$X_1 \quad X_2 \quad Z_1 \quad Z_2 \quad S \quad S$



$$X_2 + S \frac{x_2 - x_1}{l} = 0$$

$$Z_2 - S \frac{z_2 - z_1}{l} = 0$$

I

II $\quad X_1 + S \frac{x_2 - x_1}{l} = 0 \quad (z_2 - z_1)$ III

$$Z_1 + S \frac{z_2 - z_1}{l} = 0 \quad -(x_2 - x_1)$$

I $\quad X_1 + X_2 = 0$

II $\quad Z_1 + Z_2 = 0$

III $\quad X_1 (z_2 - z_1) - Z_1 (x_2 - x_1) = 0$

$$\frac{X_1}{x_2 - x_1} = \frac{Z_1}{z_2 - z_1} = \frac{\sqrt{X_1^2 + Z_1^2}}{\sqrt{(x_2 - x_1)^2 + (z_2 - z_1)^2}} = \pm \frac{R_1}{l}$$

R_1

$$X_1 = R_1 \frac{(x_2 - x_1)}{l}$$

$$Z_1 = R_1 \frac{(z_2 - z_1)}{l}$$

[illegible]

[illegible]

[illegible]

$$\left(X_1 + S\frac{x_2-x_1}{l}\right)\delta x_1 + \left(Z_1 + S\frac{z_2-z_1}{l}\right)\delta z_1 +$$

$$+\left(X_2 - S\frac{x_2-x_1}{l}\right)\delta x_2 + \left(Z_1 - S\frac{z_2-z_1}{l}\right)\delta z_2 = 0$$

[illegible]

[illegible]

[illegible]

$$(x_2-x_1)^2+(z_2-z_1)^2 = l^2$$

$$(x_2-x_1)(\delta x_1-\delta x_2)+(z_2-z_1)(\delta z_1-\delta z_2)=0$$

[illegible]

$$X_1\,\delta x_1 + Z_1\,\delta z_1 + X_2\,\delta x_2 + Z_2\,\delta z_2 -$$

$$-\frac{S}{l}\underbrace{\left[(x_2-x_1)(\delta x_2-\delta x_1)-(z_2+z_1)(\delta z_2-\delta z_1)\right]}_{=0}=0$$

[illegible]

[illegible] δx_1 ... [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$x \; y \; z \qquad X \; Y \; Z$

$$\left.\begin{array}{l} \sum (X\,\delta x + Y\,\delta y + Z\,\delta z) = 0 \\ X_1 \ldots \\ \sum \end{array}\right\} = 0$$

[illegible]

[illegible]

$$\sum\left[\left(X - m\frac{d^2x}{dt^2}\right)\delta x + \left(Y - m\frac{d^2y}{dt^2}\right)\delta y + \left(Z - m\frac{d^2z}{dt^2}\right)\delta z\right] = 0$$

[illegible]

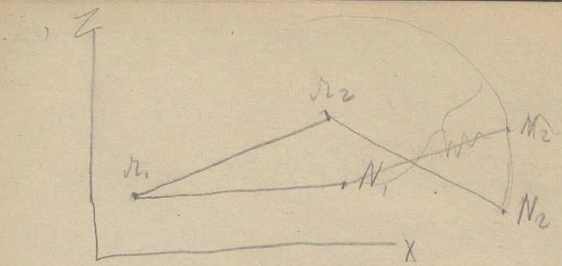
$$\sum\left[(X\,\delta x + Y\,\delta y + Z\,\delta z) - \sum m\left(\frac{d^2x}{dt^2}\delta x + \frac{d^2y}{dt^2}\delta y + \frac{d^2z}{dt^2}\delta z\right)\right] = 0$$

[illegible]

[illegible] zB. [illegible]

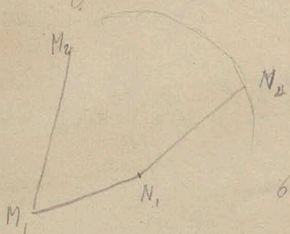
[illegible]

[illegible]


Z
M2
N1
N2
X

[illegible]

[illegible]


M2
N2
N1
M1

[illegible]

[illegible]

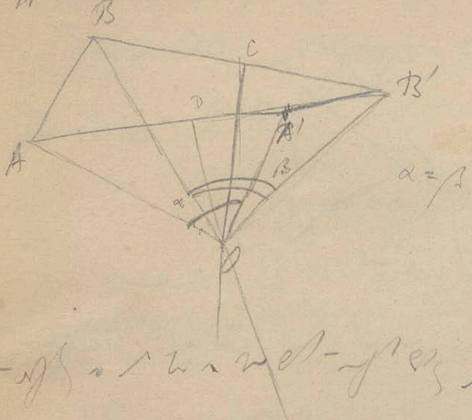
[illegible]

[illegible]

[illegible]

[illegible]

[illegible]



$BOC =$

$BOD + DOC = COA + AOB$

$AOB + BOD = DOC + COA$

$AOB - DOC = DOC - AOB$

$\sphericalangle AOB = \sphericalangle DOC$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]?

[illegible] Coord. [illegible]

[illegible]

[illegible]?

[illegible]

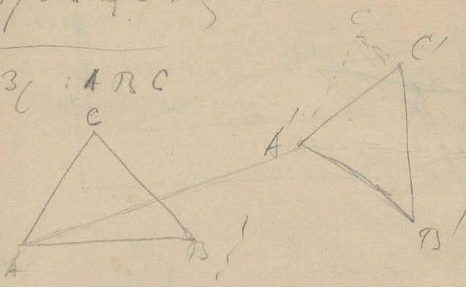
[illegible] [Tetraeder] [illegible] [illegible] [illegible]

[illegible]

~~[illegible]~~ [illegible]

[illegible]

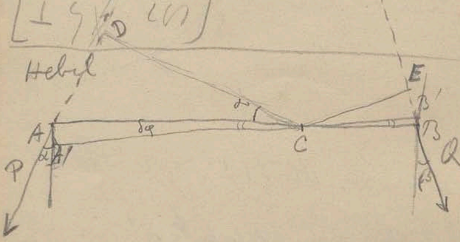
[illegible] : ABC



[illegible] A — A' [illegible] [B [illegible]]

C [illegible] C [2] [illegible] B' [illegible] A'C' [1]

[⊥ [illegible]]



[illegible] C [illegible]

$\delta\varphi = v.V.$

[illegible] $P\cos\alpha . AA' = \sqrt{e}/P$

$- Q\cos\beta . BB' = \sqrt{e}/Q$

$P\cos\alpha\, AA' - Q\cos\beta\, BB' = 0$ $\qquad AA' = AC\,\delta\varphi$

$P\cos\alpha . AC\,\delta\varphi - Q\cos\beta\, BC\,\delta\varphi = 0$ $\qquad BB' = BC\,\delta\varphi$

[illegible] $\delta\varphi$

$P \cos\alpha \, AC = Q \cos\beta \, BC$ [illegible]

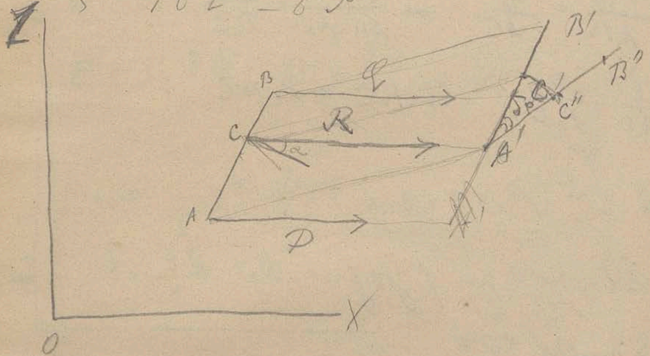
$AC \cos\alpha = DC \qquad BC \cos\beta = CE$

$P . DC = Q . EC$

[illegible]

[illegible] momento [illegible]

[illegible]



$\delta x \quad \sout{\delta y} \quad \delta z$

$P\delta x \quad R\delta x \quad Q\delta x \quad R\, C'C'' \cos\alpha \quad Q\, B'B'' \cos\alpha$

$C'C'' = AC\,\delta\varphi \qquad B'B'' = AB\,\delta\varphi$

~~$P\delta x + Q\delta x$~~

$(P+R+Q)\delta x + (R.AC + Q.AB)\cos\alpha\,\delta\varphi = 0$

$P+R+Q = 0$

$R.AC + Q.AB = 0$

[illegible shorthand] P S Q [illegible shorthand]

$$R = -(P+Q)$$

$$Q . AB = - R . AC = (P+Q) . AC$$

[illegible shorthand] C [illegible shorthand] P S Q [illegible shorthand]

[illegible shorthand]

$$\frac{P+Q}{AB} = \frac{Q}{AC} = \frac{P}{AB-AC} = \frac{P}{BC}$$

$$\frac{Q}{AC} = \frac{P}{BC} \qquad Q . BC = P . AC$$

[illegible shorthand] P S Q [illegible shorthand] Resultan[illegible]

[illegible shorthand] Res. [illegible shorthand]

[illegible shorthand]

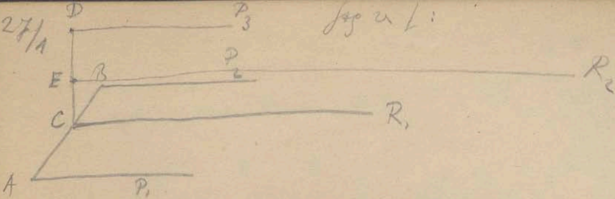
[illegible shorthand] R [illegible shorthand]

[illegible] $P+Q = 0$ $\qquad P = -Q$

$$\frac{Q}{AC} = \frac{P}{BC} \qquad \frac{Q}{AC} = \frac{-P}{BC}$$ [illegible shorthand]

[illegible shorthand] R [illegible shorthand]

27/1 [illegible]

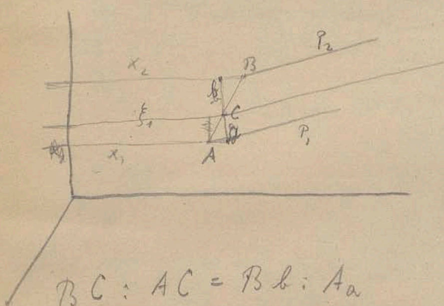


$R_1 = P_1 + P_2$

$R_1 \, AC = P_2 \, BC$

$R_2 = R_1 + P_3$

$R_1 \, CE = P_3 \, DE$ [illegible]



$BC : AC = Bb : Aa$

$= (x_2 - \xi_1) : (\xi_1 - x_1)$

$P_1 : P_2 = BC : AC$

$\frac{P_1}{P_2} = \frac{x_2 - \xi_1}{\xi_1 - x_1}$

$P_1 \xi_1 - P_1 x_1 = P_2 x_2 - P_2 \xi_1$

$(P_1 + P_2) \xi_1 = P_1 x_1 + P_1 x_2$

$\xi_1 =$ ______

[illegible]

[illegible] /3 [illegible]:

$x_1 \quad x_2 \quad x_3$

$$(P_1 + P_2)\xi_1 = P_1 x_1 + P_2 x_2$$

$$(P_1 + P_2 + P_3)\xi_2 = (P_1 + P_2)\xi_1 + P_3 x_3$$

$$= P_1 x_1 + P_2 x_2 + P_3 x_3$$

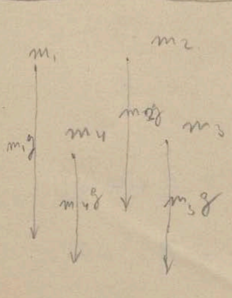
[illegible]

[illegible]

[illegible]

[illegible]

[illegible] Res. [illegible]?



[illegible]

$$(m_1 g + m_2 g + m_3 g + m_4 g)\xi =$$

$$m_1 g x_1 + m_2 g x_2 + m_3 g x_3 + m_4 g x_4$$

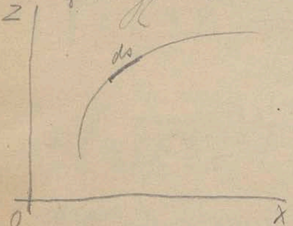
[illegible]

$$\xi \Sigma m = \Sigma m x \qquad \xi = \frac{\Sigma m x}{\Sigma m} \qquad \eta = \frac{\Sigma m y}{\Sigma m} \qquad \zeta = \frac{\Sigma m z}{\Sigma m}$$

[illegible] continuant.

[illegible] cont. [illegible] dim.

[illegible]



[illegible]

$\mu\, ds$

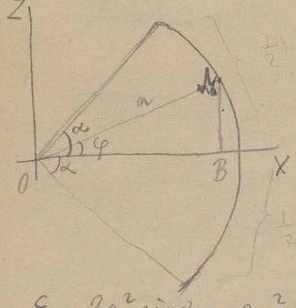
$\mu =$ [illegible]

$$\xi = \frac{\Sigma \mu\, ds\, x}{\Sigma \mu\, ds} = \frac{\int \mu\, ds\, x}{\int \mu\, ds} = \frac{\mu \int x\, ds}{\mu \int ds}$$ [illegible]

[illegible]

$$\xi = \frac{\int z\, ds}{s}$$

[illegible]



$x = a \cos\varphi$

$ds = a\, d\varphi$

$x\, ds = a^2 \cos\varphi\, d\varphi$

$$\int x\, ds = \int_{-\alpha}^{+\alpha} a^2 \cos\varphi\, d\varphi = \Big| a^2 \sin\varphi \Big|_{-\alpha}^{\alpha}$$

$$= 2a^2 \sin\alpha$$

$$\xi = \frac{2a^2 \sin\alpha}{s} = \frac{2a^2 \sin\alpha}{2a\alpha} = \frac{a \sin\alpha}{\alpha}$$

Halbkreis $\alpha = \frac{\pi}{2}$

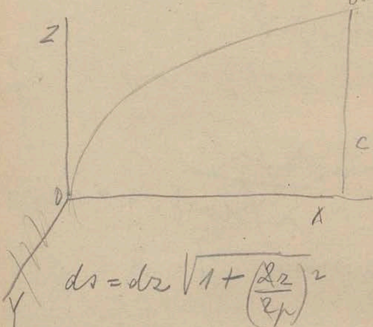
$$\xi = \frac{a}{\frac{\pi}{2}} = \frac{2a}{\pi}$$



$2a \sin\alpha = c$ = [illegible]

$$\xi = \frac{ac}{s}$$

Parabel



$$z^2 = 2px$$

$$ds^2 = dx^2 + dz^2$$

$$ds = dx\sqrt{1+\left(\frac{dz}{dx}\right)^2}$$

od.

$$ds = dz\sqrt{1+\left(\frac{dx}{dz}\right)^2}$$

$$ds = dz\sqrt{1+\left(\frac{2z}{2p}\right)^2}$$

$$\frac{\int x\,ds}{s} = \xi$$

$$\xi = \frac{\int z\,ds}{s} \qquad \int z\,ds = \int z\,dz\sqrt{1+\frac{z^2}{p^2}}$$

$$= \frac{1}{p}\int \sqrt{p^2+z^2}\, z\,dz$$

$$= \frac{1}{p}\,\frac{1}{3}\left(p^2+z^2\right)^{\frac{3}{2}}\Big|_0^c$$

$$\int z\,ds = \frac{1}{3p}\left[\left(p^2+c^2\right)^{\frac{3}{2}}\right] - \frac{1}{3p}\,p^3 =$$

$$\int z\,ds = \frac{1}{3p}(p^2+c^2)^{\frac{3}{2}} - p^3$$

$$\int ds = s = \frac{1}{p}\int dz\sqrt{p^2+z^2} = \frac{1}{p}\left(z\sqrt{p^2+z^2} - \int\frac{z^2\,dz}{\sqrt{p^2+z^2}}\right)$$

$$= \frac{1}{p}\left(z\sqrt{p^2+z^2} + p^2\int\frac{dz}{\sqrt{p^2+z^2}} - \int\sqrt{p^2+z^2}\,dz\right)$$

$$2s = \frac{1}{p}\left[z\sqrt{p^2+z^2} + p^2 \ln\left(z+\sqrt{p^2+z^2}\right)\right]$$

$z=c \mid z=0$

$$s = \frac{1}{2p}\left[c\sqrt{p^2+c^2} + p^2 \ln\frac{c+\sqrt{p^2+c^2}}{p}\right]$$

$$\xi = \frac{\int z\,ds}{s}$$

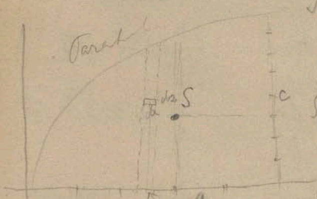
[illegible]

$$\xi = \frac{\sum m x}{\sum m} \qquad m = dF.\mu$$

$$\xi = \frac{\int x\,dF}{\int dF}$$

[illegible]

$$z^2 = 2px$$



$$\xi = \frac{\iint x\,dx\,dz}{\int dx\,dz} \qquad \zeta = \frac{\int z\,dx\,dz}{\int dx\,dz}$$

x c. z mit.

$$\int dx\,dz = \int dx\int_0^z dz = \int dx\,z$$

$$= \int\sqrt{2p}\sqrt{x}\,dx = \chi$$

$= \frac{2}{3}\sqrt{2p}\, x^{\frac{3}{2}}\Big|_0^a \quad = \frac{2}{3}\, a\sqrt{2pa}$

$= \frac{2ac}{3}$

$F = \frac{2ac}{3}$

$\iint x\,dx\,dz = \int x\,z\,dx$

$= \sqrt{2p}\int x^{\frac{3}{2}}dx = \frac{2}{5}\sqrt{2p}\; x^{\frac{5}{2}}\Big|_0^a$

$= \frac{2}{5}\, a^2\sqrt{2pa} = \frac{2}{5}\, a^2 c$

$\xi = \dfrac{\frac{2}{5}\, a^2 c}{\frac{2}{3}\, ac} = \frac{3}{5}\, a$

$\zeta = \underline{\int z\,dx\,dz} \qquad \int z\,dz = \frac{z^2}{2} = p\int x\,dx$

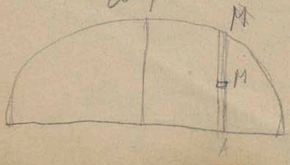
$= p\frac{x^2}{2}\Big|_0^a = p\frac{a^2}{2}$

$\zeta = \dfrac{p\frac{a^2}{2}}{\frac{2ac}{3}} = \frac{3pa}{4c} = \frac{3c^2}{8c} = \frac{3c}{8}$

---

8. Uf:

Ellipse



$\int x\,dF \qquad \frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$

$\int x\,z\,dx \qquad z = c\sqrt{1 - \frac{x^2}{a^2}}$

$\int x\,\frac{c}{a}$

$\frac{c}{a}\int \sqrt{1 - \frac{x^2}{a^2}}\; x\,dx = \frac{c}{a}\,(a^2 - x^2)^{\frac{3}{2}}\,\frac{1}{3}$

$$\frac{c}{3a}(a^2-x^2)^{\frac{3}{2}}\Big|_p^a \qquad \frac{c}{3a}(a^2-p^2)^{\frac{3}{2}} = \int x\,dF$$

$$\iint z\,dx\,dz = \int \frac{z^2}{2}dx = \frac{1}{2}c^2\int 1-\frac{x^2}{a^2}\,dx$$

$$= \frac{c^2}{2}\left(x-\frac{x^3}{3a^2}\right)\Big|_p^a$$

$$= \frac{c^2}{2}\left(\frac{2a}{3}-p+\frac{p^3}{3a^2}\right)$$

$$F = \int z\,dx = \int c\,dx\sqrt{1-\frac{x^2}{a^2}} = \frac{c}{a}\int dx\sqrt{a^2-x^2}$$

$$F = \frac{c}{a}\left[x\sqrt{a^2-x^2}+\int\frac{x^2dx}{\sqrt{a^2-x^2}}\right] =$$

$$= \frac{c}{a}\left[x\sqrt{a^2-x^2}+a^2\int\frac{dx}{\sqrt{a^2-x^2}}-\int\frac{dx}{\sqrt{a^2-x^2}}\right]$$

$$F = \frac{c}{2a}\left[x\sqrt{a^2-x^2}+a^2\arcsin\frac{x}{a}\right]\Big|_p^a$$

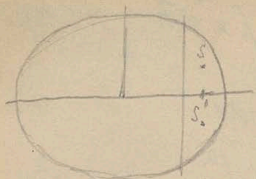
$$= \frac{c}{2a}\left[p\sqrt{a^2-p^2}+a^2\arcsin\frac{p}{a}\right] \quad \frac{c}{2a}\left[a^2\frac{\pi}{2}-p\sqrt{a^2-p^2}-a^2\arcsin\frac{p}{a}\right]$$

$p=0$ [Quadrant]

$$F = \frac{\pi a c}{4} \qquad \xi = \frac{\frac{ca^2}{3}}{\frac{\pi a c}{4}} = \frac{4a}{3\pi}$$

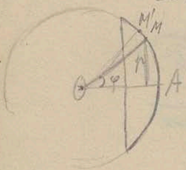
$$\int x\,dF = \frac{ca^2}{3} \qquad \zeta = \frac{\frac{ac^2}{3}}{\frac{\pi a c}{4}} = \frac{4c}{3\pi}$$

$$\int z\,dF = \frac{ac^2}{3}$$



[illegible]

z.B. Kugelschale



$MP = a \sin\varphi$

$2\pi a \sin\varphi$

$2\pi a \sin\varphi \, a\, d\varphi$ = [illegible] Zone

$$2\pi a^2 \int \sin\varphi \, d\varphi \Big|_0^{\varphi_1} = -2\pi a^2 \cos\varphi \Big|_0^{\varphi_1} = F$$

$$= 2\pi a^2 (1 - \cos\varphi_1)$$

$$\int F dx = \int 2\pi a^2 \sin\varphi \, d\varphi \cdot a \cos\varphi = 2\pi a^3 \frac{\sin^2\varphi}{2} =$$

$$= \pi a^3 \sin^2\varphi_1$$

Halbk.

$$\xi = \frac{\pi a^3}{2\pi a^2} = \frac{a}{2}$$

29/1 1 ~ Schwerpunktsbest.

$$\xi = \frac{\sum m x}{\sum m} \qquad dV = \text{Vol.-el.} \qquad \rho = \text{Dichte}$$

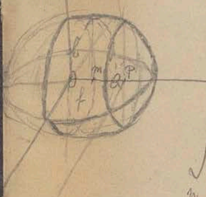
$$\rho\, dV = dm$$

$$\xi = \frac{\iiint \rho x\, dV}{\rho V} = \frac{\iiint x\, dV}{V}$$

~ Jr ρ Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$f\,dx$ – elliptische Platte



$$\int_p^a f\,dx \qquad \int x\, dV = \int_p^a f x\, dx$$

OP = p   ... Ellipse

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 - \frac{x^2}{a^2} \qquad [x = \text{const}]$$

$$\frac{y^2}{b^2\left(1-\frac{x^2}{a^2}\right)} + \frac{z^2}{c^2\left(1-\frac{x^2}{a^2}\right)} = 1$$

$$\frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1$$

$$\beta = b\sqrt{1-\frac{x^2}{a^2}} \qquad \gamma = c\sqrt{1-\frac{x^2}{a^2}}$$

$$f = \pi\beta\gamma = \pi bc\left(1-\frac{x^2}{a^2}\right)$$

$$V = \int f\,dx = \pi bc \int_p^a \left(1-\frac{x^2}{a^2}\right) dx = \pi bc \left(x - \frac{x^3}{3a^2}\right)\Big|_p^a =$$

$$= \pi bc \left[\frac{2a}{3} - p + \frac{p^3}{3a^2}\right]$$

$$\int x\,dV = \pi bc \int_p^a \left(1 - \frac{x^2}{a^2}\right) x\,dx$$

$$= \pi bc \left[\frac{x^2}{2} - \frac{x^4}{4a^2}\right]_p^a = \pi bc \left[\frac{a^2}{4} - \frac{p^2}{2} + \frac{p^4}{4a^2}\right]$$

$\xi =$

zB. [illegible] halbe Ell.

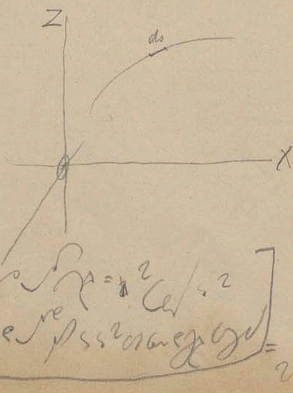
$p = 0$

$V = \frac{2\pi abc}{3}$ $\int x\,dV = \frac{\pi a^2 bc}{4}$

$\xi = \frac{3a}{8}$ [illegible] $\zeta = \frac{3c}{8}$ $\eta = \frac{3b}{8}$

[illegible] [20c]

Guldin'sche Regel



$\xi = \frac{\int z\,ds}{s}$

$2\pi z\,ds =$ [illegible]

[illegible]

$O = \int 2\pi z\,ds = 2\pi \int z\,ds$

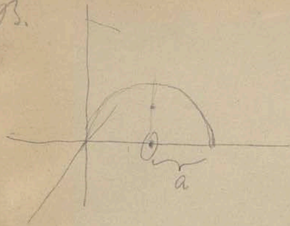
$= 2\pi s \xi = 2\pi \xi s$

[illegible]

[illegible] = [illegible]

33.

$\zeta = \frac{2a}{\pi}$



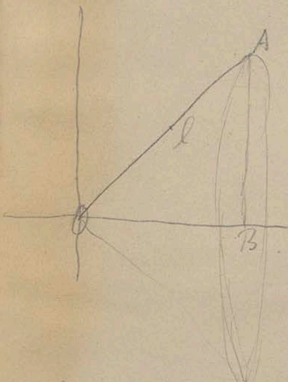
$\not{4}\pi a$

$s = \not{4}\pi a$

$\not{4}\pi a \cdot 2\pi \cdot \frac{2a}{\pi} = 4a^2\pi$

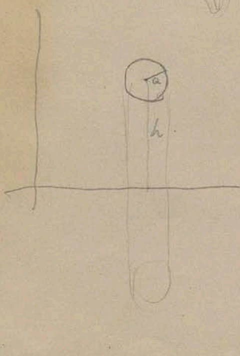
Kugel: $4\pi a^2 = 2\pi \cdot \pi a \cdot \zeta$

$$\zeta = \frac{2a}{\pi}$$

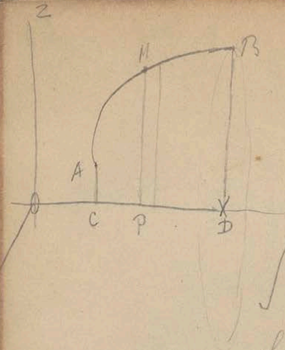


$O = 2\pi \cdot OA \cdot \frac{AB}{2}$

$= \pi l r$



$O = 2\pi a \cdot 2\pi \cdot h = 4a\pi^2 h$



$\pi z^2 \cdot dx = \text{vol.}$

$\int \pi z^2 dx =$

$\int z dx \cdot \frac{z}{2} = \int z dF = \frac{\int z dF}{F} = \xi$

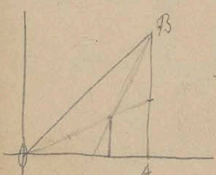
$f = \int z dx \qquad = \frac{\int \frac{z^2}{2} dx}{F}$

$\int z^2 dx = 2 \xi F$ [illegible]

$K = 2\pi \xi F =$ [illegible]

$K = \frac{4\pi a^3}{3} = 2\pi \xi \cdot \frac{\pi a^2}{2}$ [illegible]

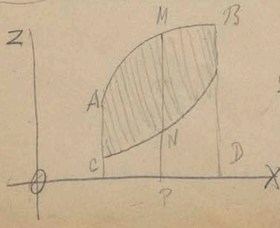
$\xi = \frac{4a}{3\pi}$



$\xi = \frac{AB}{3}$

$K = 2\pi \frac{AB}{3} \cdot \frac{OA \cdot AB}{2}$

$= \frac{\pi AB^2 \cdot OA}{3}$



$K = \int \pi z_1^2 dx - \int 2\pi z_2^2 dx$

$z_1 \; z_2 \quad = \int \pi (z_1^2 - z_2^2) dx$

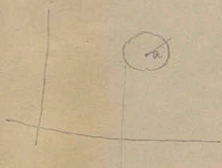
$= \pi \int (z_1 - z_2)(z_1 + z_2) dx$

$= 2\pi$

$(r_1 - r_2)\, dx = df$

$\frac{r_1 + r_2}{2} = r$

$K = 2\pi \int r\, df = 2\pi \xi f \qquad \xi = [\mathfrak{X} e, \beta]$



$K = 2\pi L \cdot \pi a^2 = 2\pi^2 L a^2$

9443

134

PAPIER-HANDLUNG

III

Mathematische Übungen

(Mechanik)

[illegible]

F. POLLY, IV. KAROLINENG.23.

$$y = x \operatorname{tg}\alpha - \frac{g}{2a^2\cos^2\alpha}x^2$$

~~$= \left(x - \frac{2\operatorname{tg}\alpha\sqrt{g}}{\sqrt{2}a\cos\alpha}\right)^2$~~

$x^2 = -2py$

$$\frac{2a^2\cos^2\alpha}{g}y = x\frac{a^2\sin 2\alpha}{g} - x^2$$

$$= -\left(x - \frac{a^2\sin 2\alpha}{2g}\right)^2 + \frac{a^4\sin^2 2\alpha}{4g^2}$$

$$\left(x - \frac{a^2\sin 2\alpha}{2g}\right)^2 = -\frac{2a^2\cos^2\alpha}{g}y + \frac{a^4\sin^2 2\alpha}{4g^2}$$

$$= -\frac{2a^2\cos^2\alpha}{g}\left[y - \frac{a^2\sin^2\alpha}{2g}\right]$$

$$h = \frac{a^2\sin 2\alpha}{2g} \qquad k = \frac{a^2\sin^2\alpha}{2g}$$

$$\frac{x}{\cos^2\alpha} - \frac{gx^2\sin\alpha}{a^2\cos^3\alpha} = 0$$

$$-1 + \frac{gx}{a^2}\operatorname{tg}\alpha = 0$$

$$\operatorname{tg}\alpha = \frac{a^2}{gx}$$

$$y = \frac{a^2}{g} - \frac{gx^2\left(1 + \frac{a^4}{g^2x^2}\right)}{2a^2} = \frac{a^2}{g} - \frac{g^2x^2 + a^4}{2a^2g} =$$

$$= \frac{2a^4 - g^2x^2 - a^4}{2a^2g} = \frac{a^2}{2g} - \frac{gx^2}{2a^2}$$

$x = a(\varphi - \sin\varphi)$ $\quad dx = a(1-\cos\varphi)\,d\varphi$

$y = a(1-\cos\varphi)$ $\quad dy = a\sin\varphi\,d\varphi$

$x' = a(\varphi - \sin\varphi - \pi)$ $\quad dx = a(1-\cos\varphi)\,d\varphi$

$y' = -a(1+\cos\varphi)$ $\quad dy = +a\sin\varphi\,d\varphi$

$$\frac{dy}{dx} = \frac{\sin\varphi}{1-\cos\varphi} = \frac{2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}}{2\sin^2\frac{\varphi}{2}} = \frac{1}{\operatorname{tg}\frac{\varphi}{2}} = \sqrt{1+\cos^2\tfrac{\varphi}{2}} =$$

$$= \sqrt{1+\frac{1+\cos\varphi}{2}} = \sqrt{1+\frac{y'}{2a}} = \sqrt{\frac{2a-y}{2a}}$$

---

54).

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \qquad \frac{dy}{dt} = \frac{cb^2}{a^2}\frac{x}{y}$$

$$\frac{y}{b^2}\frac{dy}{dt} - \frac{x}{a^2}\frac{dx}{dt} = 0 \qquad = \frac{cb^2}{a^2}\frac{x}{\frac{b}{a}\sqrt{a^2+x^2}}$$

$$\frac{y}{b^2}\frac{dy}{dt} - \frac{cx}{a^2} = 0 \qquad = \frac{cb}{a}\frac{x}{\sqrt{a^2+x^2}}$$

$$\frac{d^2y}{dt^2} = \frac{cb}{a}\,\frac{\sqrt{a^2+x^2} - \frac{x^2}{\sqrt{a^2+x^2}}}{a^2+x^2}\,\frac{dx}{dt}$$

I. $\gamma = \dfrac{cb}{a}\dfrac{ac}{(a^2+x^2)^{\frac{3}{2}}} = \dfrac{c^2b}{(a^2+x^2)^{\frac{3}{2}}} = \dfrac{c^2b^4}{a^3}\dfrac{1}{y^3}$

II. $v_y = \dfrac{cb}{a}\dfrac{x}{\sqrt{a^2+x^2}}$ $\quad v_x = c$

$$v = \sqrt{c^2 + \frac{c^2b^2}{a^2}\frac{x^2}{a^2+x^2}} = \sqrt{\frac{x^2(a^2+b^2)c^2 + a^4c^2}{a^2+x^2}}\,\frac{1}{a}$$

40). $v = c + ks$

$$\frac{ds^2}{dt^2} = K = k(c+ks)$$

$$l(c+ks) = kt + \text{const}$$

$$lc = 0 + \text{c}$$

$$t = \frac{l\frac{c+ks}{c}}{k} \qquad \text{cl.}$$

75). $\frac{dx}{dt} = c \quad Y = \frac{k^2}{y^3} \quad X = 0$

$$m\frac{d^2y}{dt^2} = -\frac{k^2}{y^3}$$

$$m\left(\frac{dy}{dt}\right)^2 = +\frac{k^2}{y^2} + \text{const}$$

$$0 = \frac{k^2}{h^2} + \text{const}$$

$$m\left(\frac{dy}{dt}\right) = \sqrt{\frac{k^2}{y^2} - \frac{k^2}{h^2}} = \sqrt{\frac{h^2k^2 - k^2y^2}{h^2y^2}} =$$

$$= \frac{k\sqrt{(h^2-y^2)}}{h^2y^2}$$

$$m\frac{dy\, h\sqrt{y^2}}{k\sqrt{(h^2-y^2)}} = dt$$

~~$t = \frac{2mk^2}{k^2}\int \frac{y^2}{h^2-y^2}dy$~~

~~$h^2 - y^2 = z^2$~~

~~$\frac{y^2}{h^2-y^2} = z^2 \quad y = \frac{hz}{\sqrt{1+z^2}}$~~

$$\cancel{dy = h\frac{\sqrt{1+z^2} - \frac{z^2}{\sqrt{1+z^2}}}{1+z^2} = h\frac{1}{(1+z^2)^{\frac{3}{2}}}}$$

$$= \frac{1}{2}\int\left(-\frac{y}{h+y} + \frac{y}{h-y}\right)dy$$

$h+y = z \qquad h-y = z' \qquad dy = dz = -dz'$

$y = z-h \qquad y = h-z'$

$$\frac{1}{2}\int \frac{h-z}{z}dz - \frac{h-z'}{z'}dz'$$

$$= \frac{1}{2}\int\left(\frac{h}{z}-1\right)dz + \left(1-\frac{h}{z'}\right)dz'$$

$$= \frac{1}{2}\left\{h\, l\, z - z + z' - h\, l\, z'\right\}$$

$$= \frac{1}{2}\left\{h\, l(h+y) - h - y + h - y - h\, l(h-y)\right\}$$

$$t = \frac{mh^2}{k^2}\left[h\, l\frac{h+y}{h-y} - 2y\right] + \text{const}$$

$$0 = \frac{mh^2}{k^2}\left[h\, l\right.$$

$$\frac{mh}{k}\int\frac{y}{\sqrt{h^2-y^2}} = dt$$

$$t = -\frac{k}{mk}\sqrt{1-\frac{y^2}{h^2}} + \text{const}$$

$$t = \frac{k}{mh}\left[\sqrt{1-\frac{y^2}{h^2}}\right]$$

$$= \frac{k}{mh^2}\sqrt{h^2-y^2}$$

~~$\left(\frac{2mht}{k}-1\right)^2 = 1-\frac{y^2}{h^2}$~~

$$\frac{x}{c} = \frac{k}{2k}\left[\sqrt{1-\frac{y^2}{h^2}}\right]$$

$$\frac{k^2x^2}{k^2c^2} = 1-\frac{y^2}{h^2}$$

etc.

94). $\frac{dx}{dt} = L\sqrt{x}$ $\qquad \frac{d^2x}{dt^2} = \frac{L}{2\sqrt{x}}\frac{dx}{dt} = \frac{L^2}{2}$

$\frac{dy}{dt} = M\sqrt{y}$ $\qquad \frac{d^2y}{dt^2} = \frac{M}{2\sqrt{y}}\frac{dy}{dt} = \frac{M^2}{2}$

$\frac{dz}{dt} = N\sqrt{z}$ $\qquad \frac{d^2z}{dt^2} = \frac{N}{2\sqrt{z}}\frac{dz}{dt} = \frac{N^2}{2}$

$2\sqrt{x} = Lt$

$2\sqrt{y} = Mt$

$2\sqrt{z} = Nt$

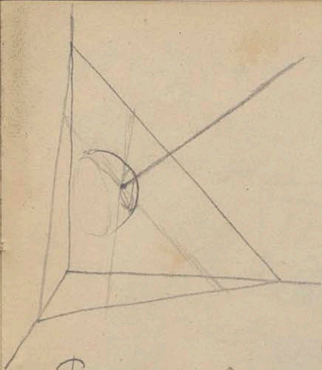
$4x M^2 - 4y L^2 = 0$

$4y N^2 - 4z M^2 = 0$

$\cos^2\lambda + \cos^2\mu + \cos^2\nu = 1$

$$\cos\mu\,\frac{dy}{\sqrt{dy^2+dz^2}} + \cos\nu\,\frac{dz}{\sqrt{dy^2+dz^2}} = 0$$

$$\cos\lambda\,\frac{dx}{\sqrt{dz^2+dx^2}} + \cos\nu\,\frac{dz}{\sqrt{dz^2+dx^2}} = 0$$



~~$F(x\,y\,z)\,z=0$~~

$$F(x\,y\,z)=0$$

$$\frac{x}{m}+\frac{y}{n}+\frac{z}{p}=1$$

$$F(x,\ y\,z)=0 \qquad\qquad F(x\,y,\,z)=0$$

$$\frac{\partial F_x}{\partial y}\,dy+\frac{\partial F_x}{\partial z}\,dz=0 \qquad\qquad \frac{\partial F_y}{\partial x}\,dx+\frac{\partial F_y}{\partial z}\,dz=0$$

$$\frac{dz}{dy}=-\frac{\dfrac{\partial F_x}{\partial y}}{\dfrac{\partial F_x}{\partial z}} \qquad\qquad \frac{dz}{dx}=-\frac{\dfrac{\partial F_y}{\partial x}}{\dfrac{\partial F_y}{\partial z}}$$

~~$(z-z_1)=\frac{dz}{dy}(y-y_1)$~~

$$\cos\lambda\cos\alpha_1+\cos\mu\cos\beta_1+\cos\nu\cos\gamma_1=0$$

$$\cos\lambda\cos\alpha_2+\cos\mu\cos\beta_2+\cos\nu\cos\gamma_2=0$$

$$\cos\alpha_1=0 \qquad \cos\beta_1=\frac{1}{\sqrt{1+\left(\frac{dz}{dy}\right)^2}} \qquad \cos\gamma_1=\frac{1}{\sqrt{1+\left(\frac{dy}{dz}\right)^2}}$$

$$\cos\alpha_2=\frac{1}{\sqrt{1+\left(\frac{dz}{dx}\right)^2}} \qquad \cos\beta_2=0 \qquad \cos\gamma_2=\frac{1}{\sqrt{1+\left(\frac{dx}{dz}\right)^2}}$$

$$\cos^2\nu = 1 - \cos^2\mu - \cos^2\lambda =$$

$$= \cos^2\mu^2 \left(\frac{dy}{dz}\right)^2$$

$$= \cos^2\lambda \left(\frac{dx}{dz}\right)^2$$

$$1 - \cos^2\mu - \cos^2\lambda = \cos^2\mu \left(\frac{dy}{dz}\right)^2$$

$$1 - \cos^2\mu - \cos^2\lambda = \cos^2\lambda \left(\frac{dx}{dz}\right)^2$$

$$\cos^2\lambda = 1 - \cos^2\mu - \cos^2\mu \left(\frac{dy}{dz}\right)^2 = \frac{1 - \cos^2\mu}{1 + \left(\frac{dx}{dz}\right)^2}$$

$$1 - \cos^2\mu \left[1 + \left(\frac{dy}{dz}\right)^2\right]$$

$$1 + \left(\frac{dx}{dz}\right)^2 - \cos^2\mu \left[1 + \left(\frac{dy}{dz}\right)^2\right]\left[1 + \left(\frac{dx}{dz}\right)^2\right] = 1 - \cos^2\mu$$

$$\cos^2\mu = \frac{\left(\frac{dx}{dz}\right)^2}{\left[1 + \left(\frac{dy}{dz}\right)^2\right]\left[1 + \left(\frac{dx}{dz}\right)^2\right] - 1}$$

$$\cos\mu = \frac{dx}{\sqrt{(dz^2 + dy^2)(dz^2 + dx^2) - dz^4}}$$

$$\cos\mu = \frac{-\frac{\partial F}{\partial y}}{\sqrt{\left[1 + \left(\frac{\frac{\partial F}{\partial z}}{\frac{\partial F}{\partial y}}\right)^2\right]\left[1 + \left(\frac{\frac{\partial F}{\partial z}}{\frac{\partial F}{\partial x}}\right)^2\right] - 1}}$$

$$\cos^2\nu\left(\frac{dz}{dx}\right)^2+\cos^2\nu\left(\frac{dz}{dy}\right)^2+\cos^2\nu=1$$

$$\cos\nu=\frac{1}{\sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}}$$

$$=\frac{1}{\sqrt{1+\left(\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}\right)^2+\left(\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}\right)^2}}=$$

$$=\frac{\frac{\partial F}{\partial z}}{\sqrt{\left(\frac{\partial F}{\partial x}\right)^2+\left(\frac{\partial F}{\partial y}\right)^2+\left(\frac{\partial F}{\partial z}\right)^2}}$$

43). $\frac{ds}{dt}=K\sqrt{\sqrt{a+s^2}-s-b+c}$

$$\frac{d^2s}{dt^2}=K\frac{\frac{s}{\sqrt{a+s^2}}-1}{2\sqrt{\sqrt{a+s^2}-s-b+c}}\frac{ds}{dt}=$$

$$=\frac{K^2}{2}\left(\frac{s}{\sqrt{a+s^2}}-1\right) \qquad \frac{b+ac^2}{b+av^2}=e^{(2as)}$$

~~44). $s=\frac{1}{2a}l\frac{b+ac^2}{b+av^2}$~~

~~$\frac{ds}{dt}=v=\frac{-1}{2a}\frac{b+av^2}{(b+av^2)^2}\frac{dv}{dt}=\frac{-1}{2a(b+av^2)}\frac{dv}{dt}$~~

~~$\frac{dv}{dt}=-2av(b+av^2)=-2av\frac{b+ac^2}{e^{2as}}$~~

44). $s = \frac{1}{2a} l \frac{b + ac^2}{b + av^2}$

$$\frac{ds}{dt} = \frac{-1}{2a} \frac{b+av^2}{(b+av^2)^2} 2av \frac{dv}{dt} = v$$

$$\frac{dv}{dt} = -(b + av^2) = \varphi$$

45). $s = kv^n = k\left(\frac{ds}{dt}\right)^n$

$$dt = \sqrt[n]{k} \frac{ds}{\sqrt[n]{s}}$$

$$t = \sqrt[n]{k} \frac{s^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} = \frac{n\sqrt[n]{k}\, s^{\frac{1-n}{n}}}{1-n} + C$$

~~$0 = \quad c \quad$~~

$$\frac{ds}{dt} = kn\left(\frac{ds}{dt}\right)^{n-1} \frac{d^2s}{dt^2}$$

$$\frac{d^2s}{dt^2} = \frac{\left(\frac{ds}{dt}\right)^{2-n}}{kn} = \frac{x \quad k^{\frac{n+2}{n}}}{kn\, s^{\frac{n+2}{n}}} =$$

$$= \frac{k^{\frac{2}{n}}}{n\, s^{\frac{n+2}{n}}}$$

58). $r = ae^{\theta}$

$$\frac{dr}{dt} = ae^{\theta}\frac{d\theta}{dt} = r\frac{d\theta}{dt}$$

$$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2}$$

$$= \sqrt{2}\, r\frac{d\theta}{dt}$$

$$\frac{d\frac{1}{r}}{d\theta} = \sqrt{-\frac{\mu r^2}{c^2} - \frac{1}{r^2} + 2c'}$$

$$\frac{1}{r^2} = z$$

$$\frac{1}{r} = \sqrt{z}$$

$$\frac{d\frac{1}{r}}{d\theta} = \frac{-1}{2\sqrt{z}}\frac{dz}{d\theta} = \sqrt{-\frac{\mu}{c^2 z} - z + 2c'}$$

$$2d\theta = \frac{-dz}{\sqrt{-\frac{\mu}{c^2} - z^2 + 2c'z}}$$

$$r = \frac{\frac{b^2}{a}}{1 + e\cos\theta} = \frac{b^2}{a + \dots\cos\theta}$$

$$\frac{1}{r^2} = \frac{1}{a^2} + \frac{a^2 - b^2}{a^2 b^2}\sin^2\theta$$

$$r^2 = \frac{1}{\frac{1}{a^2} + \frac{e^2\sin^2\theta}{a^2 b^2}} = \frac{a^2 b^2}{b^2 + e^2 - e^2\cos^2\theta} =$$

$$= \frac{a^2 b^2}{a^2 - e^2\cos^2\theta} = b$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad x = r\cos\varphi - e$$

$$y = r\sin\varphi$$

$$\frac{r^2\cos^2\varphi + e^2 - 2er\cos\varphi}{a^2} + \frac{r^2\sin^2\varphi}{b^2} = 1$$

$$b^2r^2\cos^2\varphi + a^2r^2\sin^2\varphi + e^2b^2 - 2eb^2r\cos\varphi = \frac{b^4}{}$$

$$a^2r^2 - e^2r^2\cos^2\varphi - 2eb^2r\cos\varphi = b^4$$

$$a^2r^2 - (er\cos + b^2)^2 = 0$$

$$r^2 - \frac{2eb^2\cos\varphi\, r}{a^2 - e^2\cos^2\varphi} = \frac{b^4}{a^2 - e^2\cos^2\varphi}$$

$$r = \frac{eb^2\cos\varphi}{a^2 - e^2\cos^2\varphi} \pm \sqrt{b^4a^2 - b^4e^2\cos^2\varphi + e^2b^4\cos^2\varphi}$$

$$= \frac{eb^2\cos\varphi \pm b^2a}{a^2 - e^2\cos^2\varphi} = b^2\frac{e\cos\varphi \pm a}{a^2 - e^2\cos^2\varphi}$$

$$= \frac{b^2}{a \mp e\cos\varphi} = \frac{p}{1 \mp \varepsilon\cos\varphi}$$

141

~~$\varphi = \frac{c^2}{r^2}\left(\frac{1}{r} + \frac{d^2\frac{1}{r}}{d\theta^2}\right)$~~ $c = r^2 \frac{d\theta}{dt}$

$\varphi = \frac{k}{r^3}$

~~$\frac{k}{r} = c\left(\frac{1}{r} + \frac{d^2\frac{1}{r}}{d\theta^2}\right)$~~

$\int \varphi\, dr = -\frac{c^2}{2}\left[\frac{1}{r^2} + \left(\frac{d\frac{1}{r}}{d\theta}\right)^2\right]$

$+\frac{2k}{r^2} = \frac{c^2}{2}\left[\frac{1}{r^2} + (\quad)^2\right] + c'$

$\frac{1}{r} = z$

$\sqrt{\frac{2k - \frac{c^2}{2}}{r^2} - c'} = \frac{d\frac{1}{r}}{d\theta}$

$\frac{dz}{\sqrt{\left(2k - \frac{c^2}{2}\right)z^2 - c'}} = d\theta = \frac{dz}{\sqrt{-c' - \left(\frac{c^2}{2} - 2k\right)z^2}}$

$= \frac{dz}{\sqrt{c'}\sqrt{1 - \left(\frac{2k - \frac{c^2}{2}}{c'}\right)z^2}}$

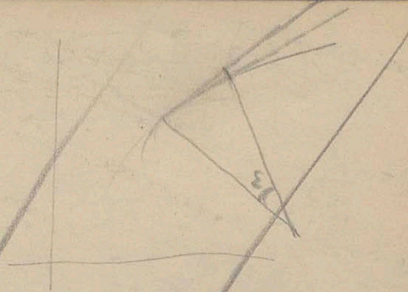
$\theta = \arcsin\left(\frac{z}{\sqrt{2k - \frac{c^2}{2}}}\right) \pm \frac{1}{r\sqrt{\quad}} = \sin\theta$

$$\frac{x^2}{a^2} + \frac{b^2}{y^2} = 0$$

~~$\rho\, ds = \varepsilon$~~

$$k = \frac{\varepsilon}{ds} = \frac{1}{\rho}$$

$$\rho = \frac{ds}{\varepsilon}$$



$$\Delta s^2 = r^2 + (r + \Delta r)^2 - 2r(r+\Delta r)\cos\varepsilon$$

$$\cos\varepsilon = \frac{r^2 + (r+\Delta r)^2 - \Delta s^2}{2r(r+\Delta r)}$$

~~$\sin\varepsilon = \sqrt{1 - (\ )^2}$~~

~~$= 4r^2(r+\Delta r)^2$~~

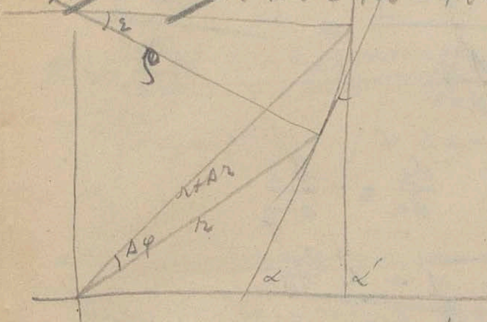
$$\sin^2\frac{\varepsilon}{2} = \frac{r^2 + 2r\Delta r - r^2 - r^2 - 2r\Delta r + \Delta r^2 + \Delta s^2}{r(r+\Delta r)}$$

$$\lim \rho = \frac{\lim \Delta s}{\lim \sin\frac{\varepsilon}{2}}$$

~~$= \frac{1}{2}\lim \sqrt{r^2\Delta\varphi^2 -}$~~

$$\varrho = \frac{\Delta s \sqrt{r^2+r\Delta r}}{r\sqrt{\Delta s^2-\Delta r^2-r^2-r\Delta r}}$$

$$4r^2\Delta s^2 - 4r^2\Delta r^2 - 4r^4 - 4r^3\Delta r = \Delta s\, r^2 + r\Delta s\Delta s^2$$



$$\rho = \frac{ds}{\varepsilon} \qquad \lim \rho = \frac{\lim \Delta s}{\lim tg\, \varepsilon}$$

$$\sin \varepsilon = \frac{tg\,\varepsilon}{\sqrt{1+tg^2\varepsilon}}$$

$$tg\, \varepsilon = tg(\alpha' - \alpha)$$

$$\frac{\Delta y}{\Delta x} \qquad = \frac{tg\,\alpha' - tg\,\alpha}{1 + tg\,\alpha\, tg\,\alpha'}$$

$$tg\, \varepsilon = \frac{\frac{\Delta y + \Delta y}{\Delta(x+\Delta x)} - \frac{\Delta y}{\Delta x}}{1 + \frac{\Delta y}{\Delta x}\frac{\Delta(y+\Delta y)}{\Delta(x+\Delta x)}} = \frac{\Delta x \Delta y + \Delta x \Delta^2 y - \Delta y \Delta x - \Delta y \Delta^2 x}{\Delta x^2 + \Delta x \Delta^2 x + \Delta y^2 + \Delta y \Delta^2 y}$$

$$= \frac{\frac{\Delta^2 y}{\Delta x^2} - \frac{\Delta y}{\Delta x}\frac{\Delta^2 x}{\Delta x^2}}{\quad} \qquad = \frac{f'(x+dx) - f'(x)}{1 + f'(x)\, f'(x+dx)} =$$

$$= \frac{f''(x)}{\quad}$$

$$\lim \rho = \lim \frac{ds\,[1+f'(x)\,f'(x+dx)]}{f'(x+dx)-f'(x)} =$$

$$= \frac{\frac{ds}{dx}\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{d^2y}{dx^2} = -\frac{b^2}{a^2}\,\frac{y - x\frac{dy}{dx}}{y^2}$$

$$= -\frac{b^2}{a^2}\,\frac{y+\frac{b^2x^2}{a^2y}}{y^2} = -\frac{b^2}{a^2}\,\frac{ab^2}{a^2y^3} =$$

$$= -\frac{b^4}{a^2y^3}$$

$$n = \frac{y}{\cos} = y\sqrt{1+\frac{b^4x^2}{a^4y^2}} = \frac{1}{a^2}\sqrt{b^4x^2+a^4y^2}$$

~~$$\rho = \frac{n^{\frac{3}{2}}}{y^{\frac{3}{2}}\,\frac{b^4}{a^2y^3}} = \frac{(yn)^{\frac{3}{2}}a^2}{b^4}$$~~

$$\rho = \left[\frac{a^4y^2+b^4x^2}{a^4y^2}\right]^{\frac{3}{2}}\frac{a^2y^3}{b^4} = \frac{\left[a^4y^2+b^4x^2\right]^{\frac{3}{2}}}{a^4b^4}$$

$$dt = \frac{r\,dr}{\sqrt{br^2 + 2\mu r - c^2}}$$

$$r = a(1 - e\cos u) \qquad b = -\frac{\mu}{a} \quad \Big| \quad c^2 = \mu a(1-e^2)$$

$$dt = \frac{a^2(1-e\cos u)\,e\sin u\,du}{\sqrt{-a\mu(1-e\cos u)^2 + 2a\mu(1-e\cos u) - \mu a(1-e^2)}}$$

$$= \frac{}{\sqrt{a\mu\left[-1 + 2e\cos u - e^2\cos^2 u + 2 - 2e\cos u - 1 + e^2\right]}}$$

$$dt = \frac{a^2(1-e\cos u)\,du}{\sqrt{a\mu}} = a\sqrt{\frac{a}{\mu}}\,(1-e\cos u)\,du$$

$$t = \frac{1}{n}(u + e\sin u)$$

~~$K = m_1 d_1 [m_2(d_2^2 + d_3^2 - 2 d_2 d_3 \cos\varphi_1)^{\frac{3}{2}} + m_2(\quad)^{\frac{3}{2}} \ldots]$~~

$u = nt + e \sin u$

$r = a(1 - e \cos u)$

$\operatorname{tg} \frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \operatorname{tg} \frac{u}{2}$

$z = x + \alpha f(z)$

$z = \varphi(\alpha)$

$z = z_0 + \alpha \left(\frac{dz}{d\alpha}\right)_0 + \frac{\alpha^2}{1 \cdot 2}\left(\frac{d^2 z}{d\alpha^2}\right) + \frac{\alpha^3}{1 \cdot 2 \cdot 3}\left(\frac{d^3 z}{d\alpha^3}\right) - $

~~$\frac{\partial z}{\partial x} = 1 + \frac{d\alpha}{dx} f(z) + \alpha \frac{df(z)}{dx}$~~ | $\frac{dz}{d\alpha}$

~~$\frac{dz}{d\alpha} = \frac{dx}{d\alpha} + f(z) + \alpha \frac{df(z)}{d\alpha}$~~ | $\frac{dz}{dx}$

~~$\frac{dz}{dx}$~~ $+ \frac{dz}{dx} f(z) =$

$\frac{\partial z}{\partial x} = 1 + \alpha \frac{\partial f(z)}{\partial x}$ | $\frac{\partial z}{\partial \alpha}$

$\frac{\partial z}{\partial \alpha} =$ ~~[illegible]~~ $f(z) + \alpha \frac{\partial f(z)}{\partial \alpha}$ | $\frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial \alpha} - f(z) \frac{\partial z}{\partial x} = 0$

$\frac{\partial z}{\partial \alpha} = f(z) \frac{\partial z}{\partial x}$

$$\left(\frac{\partial r}{\partial \alpha}\right)_0 = x$$

$$\frac{\partial^2 r}{\partial \alpha^2} = \frac{\partial f(r)}{\partial \alpha}\frac{\partial r}{\partial x} + f(r)\frac{\partial^2 r}{\partial x \partial \alpha}$$

$$\frac{\partial^2 r}{\partial \alpha \partial x} = \frac{\partial f(r)}{\partial x}\frac{\partial r}{\partial x} + f(r)\frac{\partial^2 r}{\partial x^2}$$

$$\frac{\partial^2 r}{\partial \alpha^2} = \frac{\partial f(r)}{\partial \alpha}\frac{\partial r}{\partial x} + f(r)\frac{\partial f(r)}{\partial x}\frac{\partial r}{\partial x} + [f(r)]^2\frac{\partial^2 r}{\partial x^2}$$

$$\frac{d^2 r}{d\alpha^2} = f'(r)\frac{dr}{d\alpha}\left(\frac{dr}{dx}\right) + f(r)\frac{d^2 x}{dx\, d\alpha}$$

$$\frac{d^2 r}{d\alpha\, dx} = f'(r)\left(\frac{dr}{dx}\right)^2 + f(r)\frac{d^2 r}{dx^2}$$

$$\frac{d^2 r}{d\alpha^2} = 2f(r)f'(r)\left(\frac{dr}{dx}\right)^2 + (f(r))^2\frac{d^2 r}{dx^2}$$

~~$\left(\frac{d^2 r}{d\alpha^2}\right)_0 = 2f(r)f'(r) + 2f(r)^2$~~

$$= \frac{d}{dx}\left[[f(r)]^2\frac{dr}{dx}\right] = \frac{d}{dx}\left[\frac{dx}{d\alpha}\frac{dr}{d\alpha}\right]$$

$$\left(\frac{d^2 r}{d\alpha^2}\right)_0 = \frac{d}{dx}[f(r)]^2 \quad = \frac{d}{dx}$$

$$\left(\frac{d^n z}{dx^n}\right)_0 = \frac{d^{n-1}[f(z)]^n}{dx^{n-1}}$$

$$\frac{d^n z}{dx^n} = \frac{d^{n-1}\left[f(z)^n \frac{dz}{dx}\right]}{dx^{n-1}}$$

$$\frac{d^{n+1} z}{dx^{n+1}} = \frac{d^{n-1}}{dx^{n-1}}\left[f(z)^n \frac{d^2 z}{dx\,dx} + \frac{d\,f(z)^n}{dz}\frac{d^2 z}{dx\,dx}\right]$$

$$\frac{d^{n+1} z}{dx^n\,dx} =$$

$$\frac{d\left[\varphi(z)\frac{dz}{dx}\right]}{dx} = \varphi'(z)\frac{dz}{dx}\frac{dz}{dx} + \varphi(z)\frac{d^2 z}{dx\,dx}$$

$$= \varphi'(z)\,f(z)\left(\frac{dz}{dx}\right)^2 + \varphi(z)\frac{d^2 z}{dx\,dx}$$

$$\frac{d\left[\varphi(z)\frac{dz}{dx}\right]}{dx} = \varphi'(z)\left(\frac{dz}{dx}\right)^2 + \varphi(z)\frac{d^2 z}{dx^2}$$

$$= \varphi'(z)\,f(z)\left(\frac{dz}{dx}\right)^2 + \varphi(z)\,f(z)\frac{d^2 z}{dx^2}$$

$$+ \varphi(z)\,f'(z)\left(\frac{dz}{dx}\right)^2$$

$$= \frac{d}{dx}\left[\varphi(z)\,f(z)\left(\frac{dz}{dx}\right)\right]$$

$$\frac{d^{n+1} z}{d\alpha^{n+1}} = \frac{d^{n-1}}{dx^{n+1}}\left[f(z)^n \frac{dz}{dx}\frac{dz}{dx} + \frac{d\,f(z)^n}{dz}\frac{d^2 z}{dx\,dz}\right]$$

$$= \frac{d^{n-1}}{dx^{n-1}}\left[f(z)^n f(z)\left(\frac{dz}{dx}\right)^2 + \frac{d\,f(z)^n}{dz}\left[\ \cdots\right.\right.$$

$$= \qquad + \frac{d f(z)^n}{dz} f'(z)\left(\frac{dz}{dx}\right)^2 + \frac{d f(z)^n}{dz} f(z) \frac{d^2 z}{dx^2}\Big]$$

$$= \frac{d^{n-1}}{dx^{n-1}}\left[f(z)^{n+1}\left(\frac{dz}{dx}\right)^2 + \cdots\right.$$

$$= \frac{d^n}{dx}$$

$$z = x + \alpha f(z)$$

$$z = z_0 + \left(\frac{dz}{d\alpha}\right)_0 \alpha + \left(\frac{d^2 z}{d\alpha^2}\right)_0 \frac{\alpha^2}{1.2} + \cdots$$

~~$$\frac{\partial z}{\partial x} = \frac{\partial x}{\partial x} f(z) + \alpha \frac{\partial f(z)}{\partial x}$$~~

$$z = \varphi(x, \alpha)$$

$$\frac{dz}{dx} = \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial \alpha}\frac{d\alpha}{dx}$$

$$= 1 + f(z)$$

$$z = \varphi(x, \alpha)$$

$$\frac{\partial z}{\partial x} = 1 + \alpha \frac{\partial f(z)}{\partial x}$$

$$\frac{\partial z}{\partial \alpha} = f(z) + \alpha \frac{\partial f(z)}{\partial \alpha}$$

$$\frac{dz}{d\alpha} = \frac{\partial z}{\partial x}\frac{dx}{d\alpha} + \frac{\partial z}{\partial \alpha} = \frac{dx}{d\alpha} + \alpha \frac{\partial f(z)}{\partial x}\frac{dx}{d\alpha} + f(z) + \alpha \frac{\partial f(z)}{\partial \alpha} \quad d\alpha$$

$$\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial \alpha}\frac{d\alpha}{dx} = 1 + \alpha \frac{\partial f(z)}{\partial x} + f(z)\frac{d\alpha}{dx} + \alpha \frac{\partial f(z)}{\partial \alpha}\frac{d\alpha}{dx} \quad dx$$

$$\frac{dz}{d\alpha} = \frac{dx}{d\alpha} + f(z) + \alpha \frac{d f(z)}{d\alpha} \frac{dz}{d\alpha} \frac{dz}{dx}$$

$$\frac{dz}{dx} = 1 + f(z) \frac{d\alpha}{dx} + \alpha \frac{d f(z)}{d\alpha} \frac{dz}{dx} \frac{dz}{d\alpha}$$

$$\frac{dz}{d\alpha} - \frac{dz}{d\alpha} + f(z) \left[\frac{dz}{dx} - \frac{dz}{dx}\right] +$$

$$\frac{\partial z}{\partial x} = 1 + \alpha \frac{\partial f(z)}{\partial x}$$

$$\frac{\partial z}{\partial \alpha} = f(z) + \alpha \frac{\partial f(z)}{\partial \alpha}$$

$$\alpha \frac{\partial f(z)}{\partial \alpha} + \alpha \frac{\partial f(z)}{\partial x} \frac{dx}{d\alpha} = \alpha \frac{d f(z)}{d\alpha}$$

$$\frac{\partial z}{\partial \alpha} = f(z) + \alpha \frac{d f(z)}{d\alpha} - \frac{dx}{d\alpha} \left(\frac{\partial z}{\partial x} - 1\right)$$

108).

$$\left(X - m\frac{d^2x}{dt^2}\right)\delta x + \left(Y - m\frac{d^2y}{dt^2}\right)\delta y = 0$$

$X = 0$ $\quad F(x\,y) = 0$ $\quad Y = -mg$

$$\frac{dF}{dx}\delta x + \frac{dF}{dy}\delta y = 0$$



~~$\lambda\frac{dF}{dx} - m\frac{d^2x}{dt^2}$~~ $\quad \frac{dy}{dt} = c \quad \frac{d^2y}{dt^2} = 0$

$$\lambda\frac{dF}{dx} - m\frac{d^2x}{dt^2} = 0 \qquad \frac{dx}{dt} = \frac{dy}{dt}\frac{1}{\frac{dy}{dx}} = -\frac{dy}{dt}\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}$$

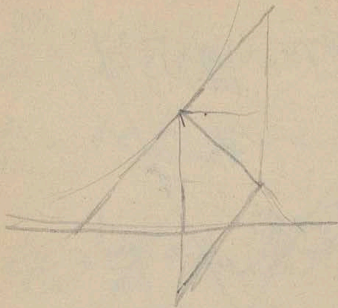
$$\lambda\frac{dF}{dy} - mg = 0$$

$$\boxed{\lambda = mg - \frac{dF}{dy}}$$

$$\frac{dx}{dt} = \frac{dy}{dt}\frac{dx}{dy} = c\frac{dx}{dy}$$

$$\frac{d^2x}{dt^2} = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d\left(\frac{dx}{dt}\right)}{dy}\frac{dy}{dt} = c^2\frac{d^2x}{dy^2}$$

$$\left.\begin{aligned}\lambda\frac{\partial F}{\partial x} &= m\frac{d^2x}{dt^2}\\ \lambda\frac{dF}{dy} &= mg\end{aligned}\right\} \quad \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{dy}{dx} = +\frac{d^2x}{dt^2}\frac{1}{g} = c^2\frac{d^2x}{dy^2}$$



$$X = \frac{N}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$

$$Y = -mg - \frac{N\frac{dy}{dx}}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$

$$m\frac{d^2x}{dt^2} = \frac{N}{\sqrt{}}$$

$$m\frac{d^2y}{dt^2} = 0 = -mg - \frac{N\frac{dy}{dx}}{\sqrt{}}$$

$$\frac{N}{\sqrt{}} = -mg\frac{dx}{dy}$$

$$\frac{d^2x}{dt^2} = -g\frac{dx}{dy}$$

$$\frac{dx}{dt} = \frac{dx}{dy}\frac{dy}{dt} = c\frac{dx}{dy}$$

$$\frac{d\frac{dx}{dt}}{dt} = \frac{d(\,)}{dy}\frac{dy}{dt} = c^2\frac{d^2x}{dy^2}$$

$$c^2\frac{d^2x}{dy^2} = -g\frac{dx}{dy}$$

$$c^2\frac{dx}{dy} = -gx$$

$$c^2\frac{dx}{x} = -g\,dy$$

2

$$c^2 l x = -gy$$

$L_1 = c.$

$_\varepsilon X \; _\varepsilon Y \; _\varepsilon Z$

$\frac{dL}{dx}\delta x + \frac{dL}{dy}\delta y + \frac{\partial L}{\partial z}\delta z = 0$

$$\int \left(\varepsilon X - \varepsilon \frac{d^2x}{dt^2}\right)\delta x + \left(\varepsilon Y - \varepsilon \frac{d^2y}{dt^2}\right)\delta y + \left(\varepsilon Z - \varepsilon \frac{d^2z}{dt^2}\right)\delta z$$

$$+ \lambda \frac{\partial L}{\partial x}\delta x + \lambda \frac{\partial L}{\partial y}\delta y + \lambda \frac{\partial L}{\partial z}\delta z +$$

$$+ X_1 \delta x_1 + Y_1 \delta y_1 + Z_1 \delta z_1 + X_2 \delta x_2 + \ldots = 0$$

$\delta x_1 = 0 \quad \delta y \ldots$

$\delta\, ds = 0$

$\frac{dx}{ds} d\delta x + \frac{dy}{ds} d\delta y \ldots = 0$

$$\lambda \frac{\partial L}{\partial x} x - \int x \, d\left(\lambda \frac{\partial L}{\partial x}\right)$$

$$\int ds \left(X - \varepsilon \frac{d^2x}{dt^2}\right)\delta x + \ldots$$

$$+ \lambda \left(\frac{dx}{ds} d\delta x + \frac{dy}{ds} d\delta y \, z \ldots\right)$$

$$+ X_1 \delta x + \ldots \quad X_2 \delta x_2 + \ldots = 0$$

$$\int \lambda \frac{dx}{ds} d\delta x = \lambda \frac{dx}{ds}\delta x \Big|_0^1 - \int \delta x \, d\left(\lambda \frac{dx}{ds}\right)$$

$$\left(X - \varepsilon \frac{d^2x}{dt^2}\right) ds - d\left(\lambda \frac{dx}{ds}\right) = 0$$

$$\left(Y - \varepsilon \frac{d^2y}{dt^2}\right) ds - d\left(\lambda \frac{dy}{ds}\right) = 0$$

$$\left(Z - \varepsilon \frac{d^2z}{dt^2}\right) ds - d\left(\lambda \frac{dz}{ds}\right) = 0$$

$X, Y, Z = 0 \quad \frac{dx}{dt} = 0 \quad \frac{dz}{dt} = 0$

$$d\left(\lambda \frac{dx}{ds}\right)=0 \qquad \lambda \frac{dx}{ds} = a$$

$$d\left(\lambda \frac{dz}{ds}\right)=0 \qquad \lambda \frac{dz}{ds} = b$$

$$\varepsilon \frac{d^2y}{dt^2} ds + d\left(\lambda \frac{dy}{ds}\right) = 0$$

$$\varepsilon \frac{d^2y}{dt^2} = -\lambda \frac{d^2y}{ds^2} = -\lambda \frac{d^2y}{dx^2}$$

$$y = F\left(x + t\sqrt{\frac{T}{\varepsilon}}\right) + f\left(x - t\sqrt{\frac{T}{\varepsilon}}\right)$$

~~$$\varepsilon \frac{dy}{dt} = \lambda \frac{dy}{dx}$$~~

$$\frac{dy}{dt} = \frac{dF}{dt}\left[\frac{dx}{dt} + \sqrt{\frac{T}{\varepsilon}}\right] + \frac{df}{dt}\left[\frac{dx}{dt} - \sqrt{\frac{T}{\varepsilon}}\right]$$

$$\frac{dy}{dx} = \frac{dF}{dx}\left[1 + \frac{dt}{dx}\sqrt{\frac{T}{\varepsilon}}\right] + \frac{df}{dx}\left[1 - \frac{dt}{dx}\sqrt{\frac{T}{\varepsilon}}\right]$$

$$= \frac{dF}{dt}\left[\frac{dt}{dx} + \left(\frac{dt}{dx}\right)^2\sqrt{\frac{T}{\varepsilon}}\right] + \frac{df}{dt}\left[\frac{dt}{dx} - \left(\frac{dt}{dx}\right)\sqrt{\ }\right]$$

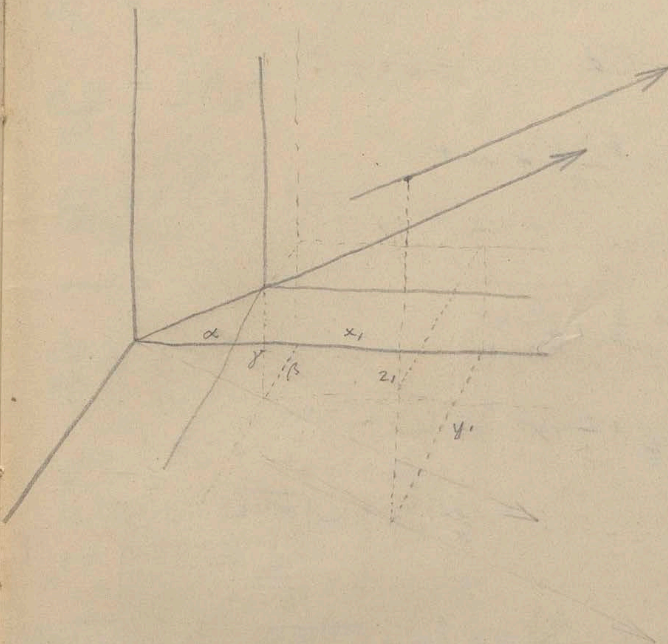
$$\frac{d^2y}{dt^2} = \frac{d^2F}{dt^2}\left[\frac{dx}{dt} + \sqrt{\ }\right] + \frac{d^2f}{dt^2}\left[\frac{dx}{dt} - \sqrt{\ }\right] +$$

$$+ \frac{dF}{dt}\left[\frac{d^2x}{dt^2}\right] + \frac{df}{dt}\left(\frac{d^2x}{dt^2}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d^2F}{dt^2}\frac{dt}{dx}\left[\frac{dt}{dx} + \left(\frac{dt}{dx}\right)\sqrt{\ }\right] + \frac{d^2f}{dt^2}\frac{dt}{dx}\left[\ \cdots\right.$$

$$+$$

$\frac{X}{Y} = \frac{x}{y}$ $\quad Xy - Yx = 0$



741). $V = \frac{c^2 y}{k^2}$

$x = 0$

$$\frac{d^2y}{dt^2} = \frac{c^2 y}{k^2} \qquad x = ct$$

$$\left(\frac{dy}{dt}\right)^2 = \frac{c^2 y^2}{k^2} + \text{const}$$

$$0 = c^2 + \text{const}$$

$$\frac{dy}{dt} = \frac{c}{k}\sqrt{y^2 - k^2}$$

$$\frac{dy}{\sqrt{y^2-k^2}} = \frac{c}{k}dt \qquad \sqrt{y^2-k^2} = z$$

$$y^2 - k^2 = z^2$$

$$y = \sqrt{k^2+z^2}$$

$$dy = \frac{z\,dz}{\sqrt{k^2+z^2}}$$

$$\int\frac{dy}{\sqrt{y^2-k^2}} = \int\frac{dz}{\sqrt{k^2+z^2}} = l\left\{\frac{z}{k} + \sqrt{1+\left(\frac{z}{k}\right)^2}\right\} = l\left(z + \sqrt{k^2+z^2}\right) + \text{const}$$

$$= l\left(y + \sqrt{y^2-k^2}\right) + \text{const}$$

$$l\left(y + \sqrt{y^2-k^2}\right) = \frac{ct}{k} + l(k)$$

$$l\,\frac{y+\sqrt{y^2-k^2}}{k} = \frac{x}{k}$$

$$\frac{y+\sqrt{y^2-k^2}}{\cancel{\sqrt{k}}} = k e^{\frac{x}{k}}$$

$$\cancel{y^2} - k^2 = \cancel{y^2} + k^2 e^{\frac{2x}{k}} - 2y\cancel{k}e^{\frac{x}{k}}$$

$$y = \frac{k}{2} \cdot \frac{e^{\frac{2x}{k}}+1}{e^{\frac{x}{k}}} = \frac{k}{2}\left(e^{\frac{x}{k}} + e^{-\frac{x}{k}}\right)$$

76). $Y = -Ce^{t} \quad X=0$

$$\frac{d^2y}{dt^2} = -Ce^{t}$$

$$\frac{dy}{dt} = -Ce^{t} + \text{Const}$$

$$c \sin\alpha = -C + \text{Const}$$

$$\frac{dy}{dt} = C - Ce^{t} + c\sin\alpha$$

$$y = Ct - Ce^{t} + ct\sin\alpha + C \quad \Big| \; x = c\cos\alpha . t$$

$$y = \left[C + c\sin\alpha\right]\frac{x}{c\cos\alpha} - Ce^{\frac{x}{c\cos\alpha}} + C \qquad t = \frac{x}{c\cos\alpha}$$

$$= \frac{C}{c}\frac{x}{\cos\alpha} + x\,tg\,\alpha - C\left(e^{\frac{x}{c\cos\alpha}} - 1\right)$$

$v = \sqrt{}$

108/. $X = -N \sin\varphi = -\frac{N y'}{\sqrt{1+y'^2}}$

$Y = -g + N \cos\varphi = -g + \frac{N}{\sqrt{1+y'^2}}$

$\frac{dy}{dt} = C\,!$

$\frac{d^2x}{dt^2} = -N \frac{dy}{dx} \frac{1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} \quad \frac{dx}{dt}$

$\frac{d^2y}{dt^2} = -g + N \frac{1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} \qquad \frac{N}{\sqrt{}} = -\frac{d^2x}{dt^2} \frac{dx}{dy}$

$\frac{d^2y}{dt^2} = -g - \frac{d^2x}{dt^2} \frac{dx}{dy}$

$\frac{d^2y}{dt^2} \frac{dy}{dt} = -g \frac{dy}{dt} - \frac{d^2x}{dt^2} \frac{dx}{dt}$

$\frac{1}{2}\left(\frac{dy}{dt}\right)^2 = -gy - \frac{1}{2}\left(\frac{dx}{dt}\right)^2$

$\left(\frac{dy}{dt}\right)^2 = -2gy - \left(\frac{dx}{dt}\right)^2$

$\left(\frac{ds}{dt}\right)^2 = -2gy$

$$\frac{N^2}{\sqrt{}} = \frac{d^2y}{dt^2} + g$$

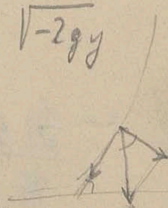
$$\frac{d^2x}{dt^2} = -\frac{dy}{dx}\left(\frac{d^2y}{dt^2} + g\right)$$

$$\frac{1}{2}\frac{d\left(\frac{ds}{dt}\right)^2}{dt} = -N\frac{dy}{dt}\frac{1}{\sqrt{}} - g\frac{dy}{dt} + N\frac{dx}{dt}\frac{1}{\sqrt{}}$$

$$\sqrt{g + N}$$

$$\frac{d^2x}{dt^2} = -N\frac{dy}{ds} \qquad \frac{ds}{dt} = \sqrt{-2gy}$$

$$\frac{d^2y}{dt^2} = -g + N\frac{dx}{ds}$$



$$\frac{d\frac{dx}{dt}}{dt} = -\frac{N}{\sqrt{-y_0}}\frac{dy}{dt}$$

$$\frac{d^2x}{dt^2} = \frac{-N}{\sqrt{-2gy}}\frac{dy}{dt}$$

$$\frac{dx}{dt} = +\frac{N\sqrt{-2gy}}{g}$$

$$\frac{d^2y}{dt^2} = -g + \frac{N}{\sqrt{-2gy_0}}\frac{dx}{dt}$$

$$\frac{d^2y}{dt^2} = -g + \frac{N^2}{g} \qquad \frac{v^2}{\rho} =$$

$$N = \frac{v^2}{\rho} + \frac{g}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$

$$\frac{dy}{dt} = c \quad \frac{d^2y}{dt^2} = 0$$

$$\frac{N}{\sqrt{}} = g \qquad N = g\sqrt{}$$

$$\frac{d^2x}{dt^2} = -\frac{dy}{dx} g \qquad \left(\frac{ds}{dt}\right)^2 = -2gy$$

$$\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}$$

$$\left(\frac{dx}{dt}\right)^2 = -2gy - c^2$$

$$\frac{d\frac{dx}{dt}}{dt} = -\frac{\frac{dy}{dt}g}{\frac{dx}{dt}} = -\frac{cg}{\frac{dx}{dt}}$$

$$\frac{d\left(\frac{dx}{dt}\right)}{dt}\frac{dx}{dt} = -cg$$

$$\left(\frac{dx}{dt}\right)^2 = 2cgt = -2gy - c^2$$

$$dx = \sqrt{2cgt}\,dt \qquad y = -\frac{2cgt + c^2}{2g}$$

$$x = \frac{2}{3}\sqrt{2cg}\,t^{\frac{3}{2}}$$

$$t = \left[\frac{3x}{2\sqrt{2cg}}\right]^{\frac{2}{3}}$$

$$2gy + c^2 = 2cg\left[\frac{3x}{2\sqrt{2cg}}\right]^{\frac{2}{3}}$$

$$\frac{dx}{dt} = \sqrt{c^2+2gy}$$

$$\frac{dx}{dy}\frac{dy}{dt} = \quad — \quad = \frac{dx}{dy}\, c$$

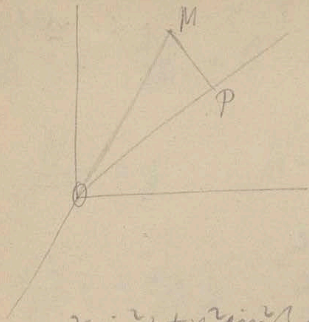
$$dx = \sqrt{1+\frac{2gy}{c^2}}\, dy$$

$$x = \frac{2}{3}\left[1+\frac{2gy}{c^2}\right]^{\frac{3}{2}}\frac{c^2}{2g}$$

~~$9x^2 = 1 + 2g$~~

$$9g^2x^2 = \left[1+\frac{2gy}{c^2}\right]^3 c^4$$

$$\left[2gy+c^2\right]^3 = 8c^2g^2\,\frac{9x^2}{8cg}$$



$$(MP)^2 = (OM)^2 - (OP)^2$$

$$= x^2 + y^2 + z^2 -$$

$$-(x\cos\alpha + y\cos\beta + z\cos\gamma)^2$$

$$= x^2\sin^2\alpha + y^2\sin^2\beta + z^2\sin^2\gamma - 2yx\cos\alpha\cos\beta -$$

$$-2xz\cos\gamma - 2yz\cos\beta\cos\gamma$$

(above $\cos\gamma$ in the first term: $\cos\alpha$)

~~S.~~

$$\Sigma x^2 = a \qquad \Sigma xy = d$$

$$\Sigma y^2 = b \qquad \Sigma yz = e$$

$$\Sigma z^2 = c \qquad \Sigma xz = f$$

$$\Sigma MP^2 = a\sin^2\alpha + b\sin^2\beta + c\sin^2\gamma - 2d\cos\alpha\cos\beta -$$

$$-2e\cos\beta\cos\gamma - 2f\cos\alpha\cos\gamma$$

~~$\cos\alpha = \frac{x}{r}$ | $\sin\alpha = \frac{?}{r}$~~

~~$a\,y^2 + b$~~ $\cos\alpha = \frac{x}{r} \quad \cos\beta = \frac{y}{r} \quad \cos\gamma = \frac{z}{r}$

$$= a\left(1 - \frac{x^2}{r^2}\right) + b\left(1 - \frac{y^2}{r^2}\right) + c\left(1 - \frac{z^2}{r^2}\right) -$$

$$-2d\,\frac{xy}{r^2} - 2e\,\frac{yz}{r^2} - 2f\,\frac{xz}{r^2} =$$

$$\frac{1}{r^2}\Big[(a+b+c)r^2 - ax^2 - by^2 - cz^2 - 2dxy - 2eyz - 2fxz\Big]$$

$$r^2 - x^2 = y^2 + z^2$$

$$= \frac{1}{r^2}\Big[a(y^2+z^2) + b(x^2+z^2) + c(x^2+y^2) - \dots\Big]$$

$$\Big[x^2(b+c) + y^2(a+c) + z^2(a+b) \dots\Big]$$

$$= r^2 \underbrace{\sum MD^2}_{\mu}$$

$$=1 \qquad r = \frac{1}{\sqrt{\mu}}$$

$x = r\cos\theta \quad y = r\sin\theta$

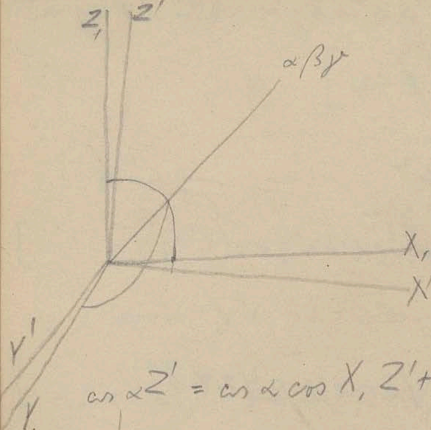
$\frac{dx}{dt} = \frac{dr}{dt} - r\sin\theta\frac{d\theta}{dt} \quad \Big| \quad \frac{dy}{dt} = \frac{dr}{dt} + r\cos\theta\frac{d\theta}{dt}$

$$\frac{d^2x}{dt^2} = \frac{d^2r}{dt^2} - \frac{dr}{dt}\sin\theta\frac{d\theta}{dt} - r\cos\theta\left(\frac{d\theta}{dt}\right)^2 - r\sin\theta\frac{d^2\theta}{dt^2}$$

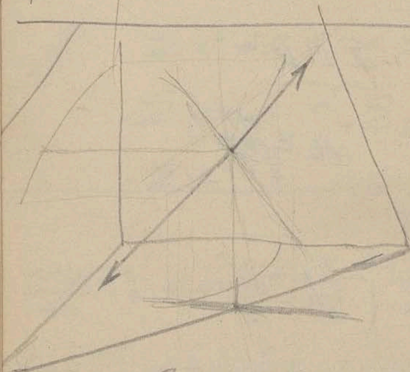
$$\frac{d^2y}{dt^2} = \frac{d^2r}{dt^2} + \frac{dr}{dt}\cos\theta\frac{d\theta}{dt} - r\sin\theta\left(\frac{d\theta}{dt}\right)^2 + r\cos\theta\frac{d^2\theta}{dt^2}$$

$r = c \quad \frac{dr}{dt} = 0 \quad \frac{d^2r}{dt^2} = 0$

$\frac{d^2x}{dt^2} = -x\omega^2 - y\frac{d\omega}{dt} \qquad \frac{d^2y}{dt^2} = -y\omega^2 + x\frac{d\omega}{dt}$



$\cos\alpha Z' = \cos\alpha\cos X_1Z' + \cos\beta\cos Y_1Z' +$ ~~$\cos\gamma\, Z_1Z'$~~



$Ax + By + Cz + D = 0$

$A'x + B'y + C'z + D = 0$

$ax + by + c = 0$

$a'x + b'z + c' = 0$

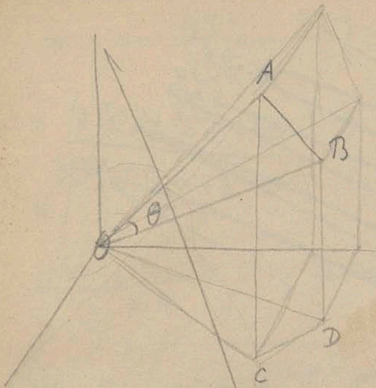
$mx + ny + p = 0$

$m'x + n'z + p' = 0$

~~$x = \frac{nc - pb}{na - mb} = x = \frac{n'c' - p'b'}{na' - m'b'}$~~

~~$y = \frac{mc - ap}{na - mb}$ $z = \frac{m'c' - a'p'}{n'a' - m'b'}$~~

$x = 0 \quad \frac{a}{m} \quad \frac{a'}{m'}$



~~cos~~

~~$\overline{AB}^2 = \overline{OA}^2 + \overline{OB}^2 - 2$~~

$$\overline{AB}^2 = \overline{OA}^2 + \overline{OB}^2 - 2\,\overline{OA}\,\overline{OB}\cos\theta$$

$$= \overline{CD}^2 + (\overline{AC} - \overline{BD})^2$$

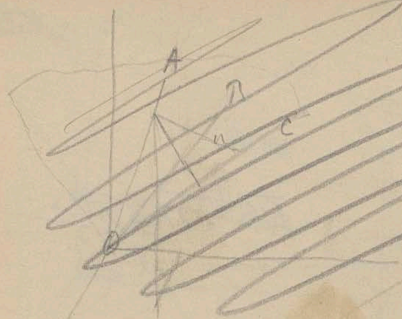
$$OA^2 + OB^2 - 2\,OA\,OB\cos\theta = CD^2 + AC^2 + BD^2 - 2AC\,BD$$

$$OA = OB = r$$

~~$2r^2(1-\cos\theta) =$~~

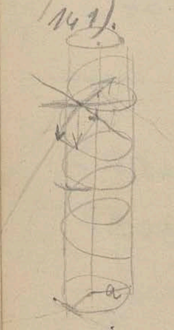
$$AC^2 = r^2\sin^2\alpha - OC^2 + r^2\cos^2\alpha$$

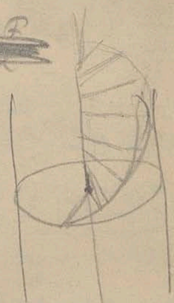
$$BD^2 =$$



$$\mathrm{tg}\,AOB = \frac{\mathrm{tg}\,AC - \mathrm{tg}\,BC}{1 + \mathrm{tg}\,AC\,\mathrm{tg}\,BC}$$

141).





$$\frac{d^2s}{dt^2} = g \sin\alpha$$

$$\frac{ds}{dt} = v = gt\sin\alpha$$

$$s = g\frac{t^2}{2}\sin\alpha \qquad s = g\frac{t^2}{2}\sin^2\alpha$$

$$y = a\sin\varphi \qquad a\varphi : s = 2a\pi : 2a\pi\,\mathrm{tg}\,\alpha$$

$$\varphi = \frac{s}{a\,\mathrm{tg}\,\alpha}$$

$$\varphi = g\frac{t^2}{2}\frac{\sin^2\alpha}{a\,\mathrm{tg}\,\alpha} = \frac{g t^2}{2a\pi}\sin 2\alpha$$

$$y = a \sin\left(\frac{gt^2}{4a}\sin 2\alpha\right)$$

$$x = a\cos\left(\frac{gt^2}{4a}\sin 2\alpha\right)$$

$$D = \sqrt{\left(\frac{v^2}{a}\right)^2 + g^2\cos^2\alpha}$$

$$= \sqrt{\left(\frac{g^2t^2\sin^2\alpha}{a}\right)^2 + g^2\cos^2\alpha} = ~~\frac{g\cos\alpha}{a}\sqrt{g^2t^4\beta^4}~~$$

$$= \frac{g\cos\alpha}{a}\sqrt{a^2 + g^2t^4\frac{\sin^4\alpha}{\cos^2\alpha}}$$

$$x = a\cos\varphi \qquad x = a\cos\left(\frac{z}{a\,tg\,\alpha}\right)$$

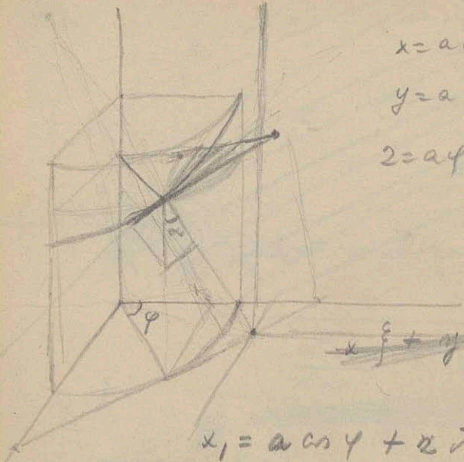
$$y = a\sin\varphi \qquad y = a\sin\left(\frac{z}{a\,tg\,\alpha}\right)$$

$$z = a\varphi\,tg\,\alpha$$

$$\frac{dx}{dz} = -a\sin\frac{z}{a\,tg\,\alpha}\cdot\frac{1}{a\,tg\,\alpha} = -\frac{\sin\left(\frac{z}{a\,tg\,\alpha}\right)}{tg\,\alpha}$$

$$\frac{dy}{dz} = \frac{\cos\left(\frac{z}{a\,tg\,\alpha}\right)}{tg\,\alpha}$$

$$\frac{dy}{dx} = -\,ctg\,\frac{z}{a\,tg\,\alpha}$$



$x = a\cos\varphi$

$y = a\sin\varphi$

$z = a\varphi \operatorname{tg}\alpha$

~~$x\xi + y\eta = a^2$~~

$x_1 = a\cos\varphi + z \operatorname{ctg}\alpha \sin\varphi$

$y_1 = a\sin\varphi - z \operatorname{tg}\alpha \cos\varphi$

$x_2 = a\cos\varphi - z \operatorname{ctg}\alpha \sin\varphi$

$y_2 = a\sin\varphi + z \operatorname{ctg}\alpha \cos\varphi$

$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - \; = -\frac{2\cos\varphi(\operatorname{ctg}\alpha + \operatorname{tg}\alpha)}{2\sin\varphi(\operatorname{ctg}\alpha + \operatorname{tg}\alpha}\left(x - \quad\right)$

$y - a\sin\varphi + z \operatorname{tg}\alpha\cos\varphi = -\operatorname{ctg}\varphi\left(x - a\cos\varphi - z\operatorname{tg}\alpha\sin\varphi\right)$

$y + x\operatorname{ctg}\varphi - \frac{a}{\sin\varphi}$ [illegible] $= 0$

$\frac{y\sin\varphi}{a} + \frac{x\cos\varphi}{a} - 1 = 0$

I $\xi = z$

$\eta = \xi \operatorname{tg} \varphi$

II $\eta \sin\varphi + \xi \cos\varphi - a = 0$

$\frac{\xi \cos\varphi}{a} + \xi \frac{\sin^2\varphi}{z} ~~+1~~ = 0$

$\xi = a \cos\varphi$

$\eta = z$

$\frac{a}{\cos\varphi} - a\cos\varphi : \frac{a}{\cos\varphi} = z : u$

~~$u = \frac{\xi z \;\; \cos\varphi}{\cos\varphi \;\; a \sin^2\varphi}$~~

I $\xi = z$

$\eta = \xi \frac{y}{x}$

II $\eta y + \xi x - a^2 = 0$

$x\xi + \xi \frac{y^2}{x} - a^2 = 0$

$\operatorname{tg} \theta_1 = \frac{g \cos\alpha}{\frac{g^2 t^2 \sin^2\alpha}{a}} = \frac{a \cos\alpha}{g t^2 \sin^2\alpha} = \frac{a \cos\alpha}{2z}$

$m_2 = z + a \frac{2z}{a \cos\alpha}$

~~$\frac{2z}{\cos\alpha} : z + \frac{2z}{\cos\alpha} = x : u$~~

~~$u = \frac{z(\cos\alpha + 2)\, x}{\cos\alpha} \frac{\cos\alpha}{2z}$~~

~~$\frac{2z}{\cos\alpha} : 2z +$~~

$\cos\theta_1 = \cos\lambda\cos\alpha + \cos\mu\cos\beta + \cos\nu\cos\gamma$

$\sin\theta_1 = \cos\lambda\cos\alpha' + \cos\mu\cos\beta' + \cos\nu\cos\gamma'$

$\cos\lambda^2 + \cos\mu^2 + \cos\nu^2 = 1$

$\cos\alpha = \cos\varphi = \frac{x}{a}$ $\qquad$ $\cos\gamma' = \cos(\alpha)$

$\cos\beta = \sin\varphi = \frac{y}{a}$

$\cos\mu = 0$

$(\xi) = 2 + a\,tg\varphi\, tg(\alpha)$

$2 + a\,tg\varphi\, tg(\alpha) : 2 = \frac{a}{\cos\varphi} + u : u$

$a\,tg\varphi\, tg\alpha : 2 = \frac{a}{\cos\varphi} : u$ $\qquad$ | $u_2 = 2$

$u_x = \frac{2}{\cos\varphi\, tg\varphi\, tg\alpha} = \frac{2}{\sin\varphi\, tg\alpha}$

$\xi_2 = 2 - a\,ctg\varphi\, tg\alpha$

$2 - a\frac{tg\alpha}{tg\varphi} : 2 = +u : u + \frac{a}{\sin\varphi}$

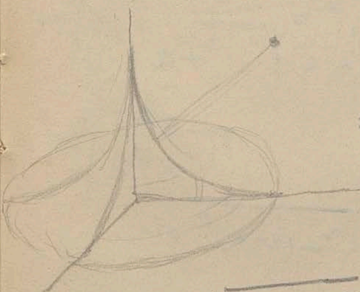
$-a\frac{tg\alpha}{tg\varphi} : 2 = -\frac{a}{\sin\varphi} : u + \frac{a}{\sin\varphi}$

$u_y = \frac{\frac{a\,2}{\sin\varphi}}{a\frac{tg\alpha}{tg\varphi}} - \frac{a}{\sin\varphi} = \frac{2}{\cos\varphi\, tg\alpha} - \frac{a}{\sin\varphi}$

$$-\frac{x}{u_x} + \frac{y}{u_y} + \frac{z}{u_z} - 1 = 0$$

$$-\frac{X}{2}\sin\varphi\,\mathrm{tg}\,\alpha + \frac{Y}{\frac{2\,\mathrm{tg}\,\alpha}{\cos\varphi} - \frac{a}{\sin\varphi}} + \frac{Z}{2} - 1 = 0$$

$$\cos\alpha' = \frac{-\sin\varphi\,\mathrm{tg}\,\alpha}{2}\,\frac{1}{\sqrt{\frac{\sin^2\varphi\,\mathrm{tg}^2\alpha}{2^2} + \frac{\cos^2\varphi\sin^2\varphi}{(2\,\mathrm{tg}\,\alpha\sin\varphi - a\cos\varphi)^2} + \frac{1}{2^2}}}$$



$$(a-x)^2 + (a-z)^2 = a^2$$

~~$(a-x)$~~

$$T = \int (\ldots)\,dx\,x^2 \cdot 2\pi$$

$$(a-z) = \sqrt{2ax - x^2}$$

$$z = a - \sqrt{2ax - x^2}$$

$$T = \int_0^a \left[a - \sqrt{2ax - x^2}\right] x^2\,dx \cdot 2\pi$$

~~$2\pi\left[\frac{ax^3}{3}\right]_0^a$~~

$$\int x^2\sqrt{2ax - x^2} =$$

$$\int x^2\sqrt{2ax-x^2} = \int x\left[(x-a)\sqrt{2ax-x^2}+a\sqrt{2ax-x^2}\right]$$

$$= \underbrace{\int x(x-a)\sqrt{2ax-x^2}\,dx}_{J_1} + \underbrace{\int ax\sqrt{2ax-x^2}\,dx}_{J_2}$$

$$J_1 = -\frac{x}{3}\left[2ax-x^2\right]^{\frac{3}{2}} + \frac{1}{3}\int\left[2ax-x^2\right]^{\frac{3}{2}}dx$$

$$\int\left[2ax-x^2\right]^{\frac{3}{2}}dx = \int 2ax\sqrt{2ax-x^2} - x^2\sqrt{2ax-x^2}$$

$$= 2J_2 - J_0$$

$$\int ax\sqrt{2ax-x^2}\,dx = a\int(x-a)\sqrt{} + a\sqrt{}$$

$$= \frac{a}{3}\left[2ax-x^2\right]^{\frac{3}{2}} + a^2 J_3$$

$$J_3 = \int\sqrt{2ax-x^2}\,dx = \int\sqrt{a^2-(a-x)^2}\,dx$$

$$= a\int\sqrt{1-\left(\frac{a-x}{a}\right)^2}\,dx$$

$$\frac{1-x}{1+x} = z^2 \qquad 1-x = z^2 + z^2 x$$

$$x = \frac{1-z^2}{1+z^2}$$

$$\int \sqrt{1-x^2}\,dx = ?$$

$$dx = \frac{-(1+z^2)z.2-(1-z^2)2z}{(1+z^2)^2} =$$

$$= \frac{-4z\,dz}{(1+z^2)^2}$$

$$\sqrt{1-x^2} = \sqrt{1-\frac{(1-z^2)^2}{(1+z^2)^2}}$$

$$= \frac{\sqrt{\cancel{1}+2z^2+\cancel{z^4}-\cancel{1}+2z^2-\cancel{z^4}}}{1+z^2} = \frac{2z}{1+z^2}$$

$$-\int \frac{2z}{1+z^2}\,\frac{4z\,dz}{(1+z^2)^2} = -8\int \frac{z^2\,dz}{(1+z^2)^3} =$$

$$= -8\left\{ \frac{-\frac{z}{4}}{(1+z^2)^2} \right.$$

$$x^2 + ax + b = 0$$

$$x^2 = -ax - b$$

$$x_1 = \sqrt{-b - a\sqrt{-b - a\sqrt{-b - a\sqrt{\cdots}}}}$$

$$x_1 = i\sqrt{b + ai\sqrt{b + ai\sqrt{b + ai\sqrt{\cdots}}}}$$

$$ax = -b - x^2$$

$$x = \frac{-b - x^2}{a} = -\frac{b}{a} - \frac{x^2}{a}$$

~~$x = \frac{-b - \left(\frac{-b - \frac{-b-a}{}}{a}\right)^2}{a}$~~

$$x_2 = -\frac{b}{a} - \frac{1}{a}\left[-\frac{b}{a} - \frac{1}{a}\left[-\frac{b}{a} - \frac{1}{a}\left[-\frac{b}{a} - \frac{1}{a}\left[\cdots\right]^2\right]^2\right]^2\right]^2$$

$$x_2 = -\frac{b}{a} - \frac{1}{a}\left[\frac{b}{a} + \frac{1}{a}\left[\frac{b}{a} + \frac{1}{a}\left[\frac{b}{a} + \frac{1}{a} \cdots\right]^2\right]^2\right]^2$$

divergent?

$$(x+5)(x+2) = 0$$

$$x^2 + 7x + 10 = 0$$

$$x_1 = i\sqrt{10 + 7i\sqrt{10 + 7i\sqrt{\cdots}}}$$

~~$\sqrt{10} = 3'1623 \cdot 7 = 22'1361$~~ = ?

100 : 61

300 : 626

144 : 632

18

~~$x_2 = -\frac{b}{a} - \frac{1}{a}[$~~

~~$x = -\frac{b}{a} = -\frac{10}{7}$ $x^2 = \frac{100}{49}$ $-b - x^2 = -\frac{590}{49}$~~

$x^2 - ax + b = 0$

$$x^2 = ax - b$$

$$x_1 = \sqrt{-b + a\sqrt{-b + a\sqrt{-b + a\sqrt{\ldots}}}}$$

$$ax = x^2 + b$$

$$x = \frac{x^2 + b}{a}$$

$$x_2 = \frac{b}{a} + \frac{1}{a}\left[\frac{b}{a} + \frac{1}{a}\left[\frac{b}{a} + \frac{1}{a} \ldots\right]^2\right]^2$$

divergent ?

$x^2 + ax - b = 0$

$$x^2 = b - ax$$

$$x_1 = \sqrt{b - a\sqrt{b - a\sqrt{b - a\sqrt{\ldots}}}}$$

$$ax = b - x^2$$

$$x = \frac{b - x^2}{a}$$

$$x_2 = \frac{b}{a} - \frac{1}{a}\left[\frac{b}{a} - \frac{1}{a}\left[\frac{b}{a} - \frac{1}{a} \ldots\right]^2\right]^2$$

$x^2 - ax - b = 0$

$$x_1 = \sqrt{b + a\sqrt{b + a\sqrt{b + a\sqrt{\ldots}}}}$$

$$x_2 = -\frac{b}{a} + \frac{1}{a}\left[-\frac{b}{a} + \frac{1}{a}\left[-\frac{b}{a} + \frac{1}{a}[\ldots]\right]^2\right]^2$$

$I \; x^2+7x+10=0 \qquad x_1=5 \mid x_2=2$

$II \; x^2-3x-10=0 \qquad x_1=-5 \mid x_2=2$

$III \; x^2+3x-10=0 \qquad x_1=5 \mid x_2=-2$

$IV \; x^2-7x+10=0 \qquad x_1=-5 \mid x_2=-2$

$I \quad x_1=? \quad x_2=?$

$II \quad \sqrt{10}=\frac{3{,}16\,23}{22\,1\,3\,6} \cdot 7$

$\sqrt{32{,}14}=5{,}67$

7 14 : 106 ... [illegible] x_1

78 : 112

$\frac{b}{a}=\frac{10}{7} \qquad \frac{100}{49\cdot 7}=\frac{100}{343} \qquad \frac{\frac{100}{49}-10}{7}=$

$=\frac{-390}{343}\neq-\frac{40}{35}\neq-\frac{8}{7}$

$\frac{64}{49\cdot 7}-\frac{10}{7}=-\frac{426}{343}\neq-\frac{6}{5}$

$\frac{36}{25\cdot 7}-\frac{10}{7}=\frac{11}{175}=\frac{1}{16}$

$\underline{1}$ — — ? [illegible]

$x_2=?$

$\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{1 \quad 2 \quad 3 \quad 4}$

III

$x_1 = ?$ $10 - 22.13 = -12.1$

24.25 -14.3

$x_2 = ?$

IV

$$\Big(\overbrace{a + b\big(a + b\big(a + b(\ldots (b)^{\frac{1}{2}}\big)^{\frac{1}{2}}\big)^{\frac{1}{2}}}^{a_1}\Big)^{\frac{1}{2}}$$

$$= a^{\frac{1}{2}} + \frac{1}{2} a^{-\frac{1}{2}} a_1 - \frac{1}{8} a^{-\frac{3}{2}} a_1^2 + \frac{1}{16} a^{-\frac{5}{2}} a_1^3 + \ldots$$

$$a_1 = b\left[a^{\frac{1}{2}} + \frac{1}{2} a^{-\frac{1}{2}} a_1 - \frac{1}{8} a^{-\frac{3}{2}} \ldots\right]$$

$$a_1^2 = b^2 [a + b a_1]$$

$$a_1^3 = b^3 \Big[a^{\frac{3}{2}} + \frac{1}{2} a^{\frac{1}{2}} a_1 - \frac{1}{8} a^{-\frac{1}{2}} a_1^2 \ldots$$

$$+ b a^{\frac{1}{2}} a_1 + \frac{1}{2} b a^{-\frac{1}{2}} a_1^2 \ldots$$

$$= b^3 \left[a^{\frac{3}{2}} + \left(\frac{1}{2} + b\right) a^{\frac{1}{2}} a_1 + \left(\frac{b}{2} - \frac{1}{8}\right) a^{-\frac{1}{2}} a_1^2 \ldots\right]$$

$$= a^{\frac{1}{2}} + \frac{1}{2} a^{-\frac{1}{2}} b \left[a^{\frac{1}{2}} + \frac{1}{2} a^{-\frac{1}{2}} \left[b a^{\frac{1}{2}} \ldots \right]\right]$$

$$- \frac{1}{8} a^{-\frac{3}{2}} b^2 \left[a + b^2 \left[a^{\frac{1}{2}} + \frac{1}{2} a^{-\frac{1}{2}} \left[b a^{\frac{1}{2}} \ldots \right]\right.\right.$$

$$dp = \rho(X\,dx + Y\,dy + Z\,dz)$$



$$\partial p\, dy\,dz = X \rho\, dx\,dy\,dz$$

$$\partial p\, dx\,dz = Y \rho\, dx\,dy\,dz$$

$$dp = \partial_x p\, dx + \partial_y p\, dy + \partial_z p\, dz$$

$$dp = \rho(X\,dx + Y\,dy + \ldots)$$

$$\rho\left(\frac{X}{R}\frac{dx}{ds} + \frac{Y}{R}\frac{dy}{ds} + \frac{Z}{R}\frac{dz}{ds}\right) = 0$$

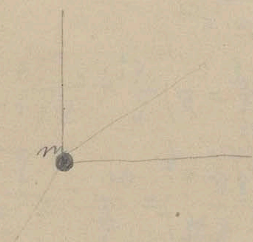
$$\cos\alpha \cos\lambda + \cos \text{ --- --- } = 0$$

$$\cos\theta = 0$$

$$\theta = 90^\circ$$

---

203. [illegible]



$$K = -\frac{mk}{x^2+y^2+z^2}$$

$$X = -\frac{mk\,x}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

~~Pro~~ $p = \alpha\rho$

$$\alpha\, d\rho = -\rho\left[mk\,\frac{x\,dx + y\,dy + z\,dz}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right]$$

~~N F. $\rho = c$~~

$$\alpha\,\frac{d\rho}{\rho} = +mk\, d\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$$

N F. $\rho = c.$ $\frac{1}{\sqrt{\cdots}} = \frac{1}{r} = \text{const}$

$$\alpha\, l\rho = \frac{mk}{\sqrt{x^2+y^2+z^2}} = \frac{mk}{r} + \text{const}$$

$$\rho = e^{\frac{mk}{\alpha r} + c}$$

---

a 2 20 p. ε R [+ ρ f ∫ e

~~$K = -\frac{mk}{x^2+y^2+z^2} + \frac{mk}{(l-x)^2+(l-y)^2+(l-z)^2}$~~

~~$\alpha\frac{d\rho}{\rho} = +mk\left[\frac{x\,dx + \cdots}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right.$~~

~~$= mk\,[$~~

$$X = -\frac{mkx}{r^3} + \frac{mk(l-x)}{r'^3}$$

$$= -\frac{mkx}{(x^2+y^2+z^2)^{\frac{3}{2}}} + \frac{mk(l-x)}{[(l-x)^2+y^2+z^2]^{\frac{3}{2}}}$$

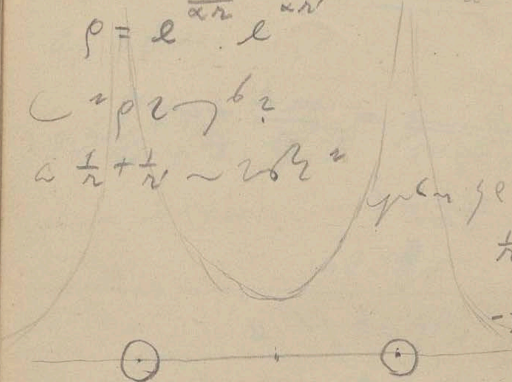
$$Y = -\frac{mky}{(x^2+y^2+z^2)^{\frac{3}{2}}} - \frac{mky}{\left[(e-x)^2+y^2+z^2\right]^{\frac{3}{2}}}$$

$$\alpha \frac{d\rho}{\rho} = mk\left[d\frac{1}{\sqrt{x^2+y^2+z^2}} + d\frac{1}{\sqrt{(e-x)^2+y^2+z^2}}\right]$$

$$\alpha \, l\rho = mk\left[\frac{1}{r} + \frac{1}{r'}\right]$$

$e - r$

$$\rho = e^{\frac{mk}{\alpha r}} \cdot e^{\frac{mk}{\alpha r'}}$$



[illegible]

$\alpha \frac{1}{r} + \frac{1}{r'} \sim$ [illegible]

[illegible] $r' = e - r$

$$\frac{1}{r} + \frac{1}{e-r}$$

$$-\frac{1}{r^2} + \frac{1}{(e-r)^2} = 0$$

$$(e-r)^2 - r^2 = 0$$

$$e^2 - 2er = 0$$

$$r = \frac{e}{2}$$

Minim

$$\frac{2}{r^3} - \frac{2}{(e-r)^3}$$

$$\frac{16}{e^3} - \frac{16}{e^3} = 0$$

$$\frac{1}{r^2(e-r)^2} = 0$$

$e = r$ = Max

$r = \infty$ = Min.

$$V = \iint 2\delta\pi \, d\delta \, dx \, e^{\frac{mk}{\alpha}\left[\frac{1}{\sqrt{x^2+\delta^2}} + \frac{1}{\sqrt{(\frac{l}{2}+x)^2+\delta^2}}\right]}$$

$$= 2\pi e^{\frac{mk}{\alpha}} \int_0^{x}\int_0^{\delta} \delta \, d\delta \, dx \left[1 + \frac{1}{\sqrt{x^2+\delta^2}} + \frac{1}{\sqrt{(\frac{l}{2}+x)^2+\delta^2}} + \right.$$

$$+ \frac{1}{x^2+\delta^2} + \frac{1}{(\frac{l}{2}+x)^2+\delta^2} + \frac{2}{\sqrt{}\sqrt{}} + \cdots$$

$$= 2\pi e^{\frac{mk}{\alpha}} \int_0^{x} dx \left[\frac{\delta^2}{2} + \sqrt{x^2+\delta^2} + \sqrt{(\tfrac{l}{2}+x)^2+\delta^2} + \right.$$

$$\left. + \tfrac{1}{2} l(x^2+\delta^2) + \tfrac{1}{2} l\left[(\tfrac{l}{2}+x)^2+\delta^2\right] + \cdots \right]_0^{\delta}$$

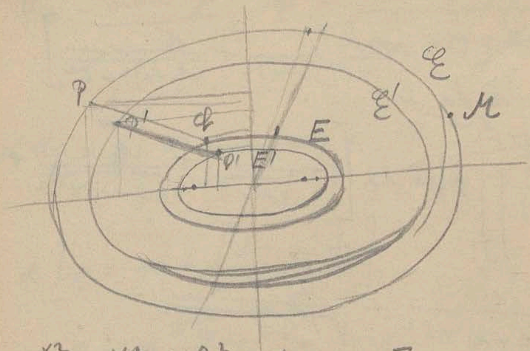
$$= 2\pi e^{\frac{mk}{\alpha}} \int_0^{x} dx \left[\frac{\delta^2}{2} + \sqrt{x^2+\delta^2} - x + \sqrt{(\tfrac{l}{2}+x)^2+\delta^2} - \frac{l}{2} - x + \right.$$

$$\left. + l(\sqrt{x^2+\delta^2}) - l(x) + l\sqrt{(\tfrac{l}{2}+x)^2+\delta^2} - l(\tfrac{l}{2}+x) \right]$$

$$= 2\pi e^{\frac{mk}{\alpha}} \left[\frac{\delta^2 x}{2} + \frac{1}{3}(x^2+\delta^2)^{\frac{3}{2}} - \frac{x^2}{2} - \frac{lx}{2} + l(x + \sqrt{x^2+\delta^2}) \right.$$

$$\left. + l\left[\tfrac{l}{2}+x+\sqrt{(\tfrac{l}{2}+x)^2+\delta^2}\right] + \cdots - l\delta - l(\tfrac{l}{2} + \sqrt{}) \right]$$

$$V_1 = 2\pi e^{\frac{mk}{\alpha}} \left[\frac{\delta^2 x}{2} - \frac{x^2}{2} + l(x + \sqrt{x^2+\delta^2}) - l\delta \cdots \right]$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad E$$

$$\frac{x'^2}{a^2\lambda^2} + \frac{y'^2}{b^2\lambda^2} + \frac{z'^2}{c^2\lambda^2} = 1 \qquad E' \text{ ähnl.}$$

$$\frac{\xi^2}{\alpha^2} + \frac{\eta^2}{\alpha^2 - e^2} + \frac{\zeta^2}{\alpha^2 - d^2} = 1 \quad \mathfrak{E} \qquad \left| \begin{array}{l} e^2 = a^2 - b^2 \\ d^2 = a^2 - c^2 \end{array} \right.$$

$$\frac{\xi'^2}{\alpha^2\lambda^2} + \frac{\eta'^2}{(\alpha^2 - e^2)\lambda^2} + \frac{\zeta'^2}{(\alpha^2 - d^2)\lambda^2} = 1$$

B. A. $[P\varphi]^2 = [P'\varphi']^2$

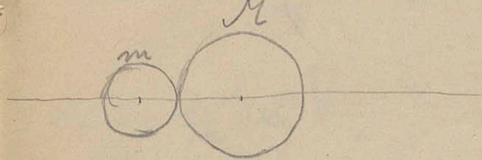
$$z = c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} \qquad z' = c\sqrt{\lambda^2 - \frac{x'^2}{a^2} - \frac{y'^2}{b^2}}$$

$$\zeta = \sqrt{(\alpha^2 - d^2)\left[1 - \frac{\xi^2}{\alpha^2} - \frac{\eta^2}{\alpha^2 - e^2}\right]} \qquad \zeta' = \sqrt{(\alpha^2 - d^2)\left[\lambda^2 - \frac{\xi^2}{\alpha^2} - \frac{\eta^2}{\alpha^2 - e^2}\right]}$$

$P\varphi^2 = \sout{x^2+y^2+z^2}$

$(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2$

$=$



$M \quad V \quad V'$

$m \quad v \quad v'$

$$\xi = \frac{MX+mx}{M+m}$$

$$\frac{d\xi}{dt} = \frac{MV+mv}{M+m} = \frac{MV'+mv'}{M+m}$$

$$M(V-V')+m(v-v')=0$$

$$V-v = v'-V'$$

$$V+V' = v'+v \qquad V' = v'+v-V$$

$$M(2V-v'-v)+m(v-v')=0$$

$$v' = \frac{2VM - Mv + mv}{M+m}$$

$$X\,dx + Y\,dy + Z\,dz = 0$$

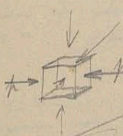
$$2\pi\rho f = \alpha$$

$$\frac{\alpha}{\lambda^3}\, x \left[\lambda - (1+\lambda^2)\operatorname{arctg}\lambda + \frac{\omega\lambda}{\alpha}\right] dx +$$

$$\frac{\alpha}{\lambda^3}\, y \left[\lambda - (1+\lambda^2)\operatorname{arct} - \frac{\omega\lambda}{\alpha}\right] dy +$$

etc.

$\lambda -$



$p + \frac{\partial p}{\partial x}\,dx$

$$\frac{\partial p}{\partial x}\,dx = X - m\frac{d^2x}{dt^2} \qquad m = \rho\,dx\,dy\,dz$$

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz$$

$$dp = X\,dx + Y\,dy + Z\,dz - \rho\,dx\,dy\,dz \left[\frac{d^2x}{dt^2} + \cdots\right]$$

$$X = \frac{\partial V}{\partial x} \quad \text{etc.}$$

~~$w\,dx\,dy + u\,dy\,dz + v\,dz\,dx = \frac{d\rho}{dt}$~~

~~$dx\,dy\,dz$~~

$$dx\,dy\,[\rho(w - w')] + dy\,dz\,[\rho(u-u')] + dz\,dx\,[\rho(v-v')] =$$

$$= \frac{d\rho}{dt}\,dx\,dy\,dz$$

$$w' = w + \frac{\partial w}{\partial z}\,dz$$

$$\frac{\partial \rho w}{\partial z} + \frac{\partial \rho u}{\partial z} + \frac{\partial \rho v}{\partial z} = \frac{\partial \rho}{\partial t}$$

$$\frac{\partial p}{\partial x}\,dx\,dy\,dz = \left(X - \frac{d^2x}{dt^2}\right)\rho\,dx\,dy\,dz$$

$$\frac{dx}{dt} = u \qquad u = f(t, x, y, z)$$

$$x = f_1(t) \quad y = f_2(t) \quad z = f_3(t)$$

$$\frac{d^2x}{dt^2} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$u = \frac{\partial \varphi}{\partial x}$$

$$\frac{\partial^2 \varphi}{\partial x\,\partial t} + \frac{\partial \varphi}{\partial x}\frac{\partial^2 \varphi}{\partial x^2} +$$

$$u = \frac{\partial \varphi}{\partial x} \quad v = \frac{\partial \varphi}{\partial y} \quad w = \frac{\partial \varphi}{\partial z}$$

$$\frac{\partial^2 \varphi}{\partial x \partial t} + \frac{\partial \varphi}{\partial x}\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial \varphi}{\partial y}\frac{\partial^2 \varphi}{\partial y \partial x} + \frac{\partial \varphi}{\partial z}\frac{\partial^2 \varphi}{\partial z \partial x} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial^2 \varphi}{\partial y \partial t} + \frac{\partial \varphi}{\partial y}\frac{\partial^2 \varphi}{\partial x \partial y} + \frac{\partial \varphi}{\partial y}\frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial \varphi}{\partial z}\frac{\partial^2 \varphi}{\partial y \partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial^2 \varphi}{\partial z \partial t} + \text{———} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

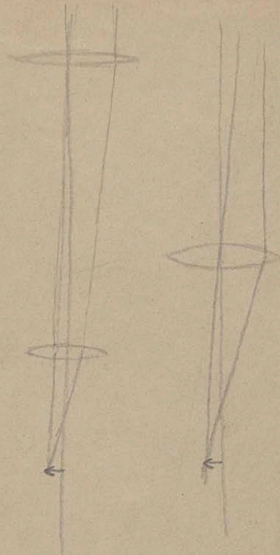
$$d\left(\frac{\partial \varphi}{\partial t}\right) + \frac{1}{2} d\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2\right] = -dV - \frac{1}{\rho}\,dp$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2}(u^2 + v^2 + w^2) + V + \int \frac{dp}{\rho} = 0$$

dx

dy

dz



$m = \frac{\delta m \cdot [6'2 \cdot 10^{-8}]^3 \; cm}{0'00062 \cdot 10^3}$

$b \gtrless 0$

$\frac{2\mu}{r_0} = v_0^2$

$v_0 = \sqrt{2 k r_0}$

$b = v_0^2 - \frac{2\mu}{r_0}$

δ cm $= [\delta] \; 0'00001$ cm $= 10^{-5}$

$\frac{\mu k}{r^2} = K = \frac{m k}{r^2}$

$K_m : K_T = \frac{m k}{10^{-10} \cdot 4} : \frac{k R^3 \pi \cdot 5'6}{R^2 \; 10^3}$

$K_m : 10^3 = \frac{m}{4 \cdot 10^{-10}} : \frac{17}{10^3} \cdot 6400 \cdot 10^5$

$K_m = \frac{10^3 [6'2 \cdot 10^{-8}]^3 \; 10^7 \cdot 0'07 \cdot 10^3}{4 \cdot 10^{-10} \cdot 0'00062 \cdot 10^3 \cdot 17 \cdot 64 \cdot 10^7 \; 10^5}$

$v_0 = \sqrt{\frac{2'38 \cdot 2}{68 \cdot 9 \cdot 10^{17}}} = \frac{1}{109}$

779
850 . 7'5
5950
4250
6400'0

36
244
3844
23064
7688
8'328